Strategy for Solving Related Rates Problems (Part 6)

At 1:00 pm ship A is 160 km West of ship B. Ship A is sailing east at 35 km/hr, and ship B is sailing North at 30 km/hr. How fast is the distance between the ships changing at 3:00 pm?

Step 1 $\frac{dx}{dt} = 35$ km/hr

$\frac{dy}{dt} = 30$ km/hr

$\frac{dD}{dt}$ = rate at which distance between the ships is changing

Step 2 x, y, D $\quad (160-x)^2 + y^2 = D^2$

Step 3

$$\frac{d}{dt}\left[(160-x)^2 + y^2\right] = \frac{d}{dt}(D^2)$$

$$\frac{d}{dt}\left[(160-x)^2\right] + \frac{d}{dt}(y^2) = \frac{d}{dt}(D^2)$$

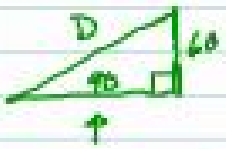
$$2(160-x)\frac{d}{dt}(160-x) + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$2(160-x)\left(0 - \frac{dx}{dt}\right) + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$-(160-x)\frac{dx}{dt} + y\frac{dy}{dt} = D\frac{dD}{dt}$$

Step 4 at 3:00 pm, $x = (35 \text{ km/hr})(2 \text{ hr}) = 70$ km

$y = (30 \text{ km/hr})(2 \text{ hr}) = 60$ km



$D^2 = 60^2 + 90^2 \Rightarrow D = \sqrt{60^2+90^2} = \sqrt{11700}$ km $= 10\sqrt{117}$ km

$$-(160-x)\frac{dx}{dt} + y\frac{dy}{dt} = D\frac{dD}{dt}$$

$$-(160-70)\,35 + (60)\,30 = 10\sqrt{117}\,\frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{-3150\ \frac{km^2}{hr} + 1800\ \frac{km^2}{hr}}{10\sqrt{117}\ km}$$

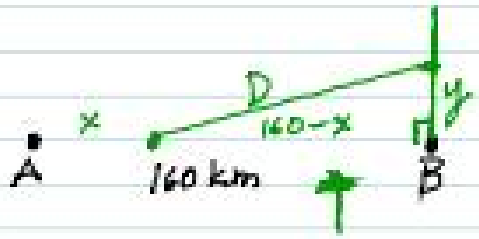
$$\frac{dD}{dt} = \frac{-1350}{10\sqrt{117}}\ \frac{km}{hr} = \frac{-135}{\sqrt{117}}\ \frac{km}{hr} \approx -12.48\ \frac{km}{hr}$$

Strategy for Solving Related Rate Problems (Part 6)

At 1:00 pm ship A is 160 km west of ship B. Ship A is sailing east at 35 km/hr, and ship B is sailing north at 30 km/hr. How fast is the distance between the ships changing at 3:00 pm?

Step 1 $\frac{dx}{dt} = 35$ km/hr

$\frac{dy}{dt} = 30$ km/hr



$\frac{dD}{dt}$ = rate at which distance between the ships is changing

Step 2 x, y, D $\quad (160-x)^2 + y^2 = D^2$

Step 3 $\frac{d}{dt}\left[(160-x)^2 + y^2\right] = \frac{d}{dt}(D^2)$

$\frac{d}{dt}\left[(160-x)^2\right] + \frac{d}{dt}(y^2) = \frac{d}{dt}(D^2)$

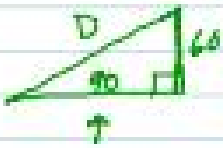
$2(160-x)\frac{d}{dt}(160-x) + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$

$2(160-x)\left(0 - \frac{dx}{dt}\right) + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$

$-(160-x)\frac{dx}{dt} + y\frac{dy}{dt} = D\frac{dD}{dt}$

Step 4 at 3:00 pm, $x = (35 \text{ km/hr})(2 \text{ hr}) = 70$ km

$y = (30 \text{ km/hr})(2 \text{ hr}) = 60$ km



$D^2 = 60^2 + 90^2 \Rightarrow D = \sqrt{60^2+90^2} = \sqrt{11{,}700}$ km $= 10\sqrt{117}$ km

$-(160-x)\frac{dx}{dt} + y\frac{dy}{dt} = D\frac{dD}{dt}$

$-(160-70)\,35 + (60)\,30 = 10\sqrt{117}\,\frac{dD}{dt}$

$\frac{dD}{dt} = \frac{-3150 \frac{\text{km}^2}{\text{hr}} + 1800 \frac{\text{km}^2}{\text{hr}}}{10\sqrt{117} \text{ km}}$

$\frac{dD}{dt} = \frac{-1350}{10\sqrt{117}} \frac{\text{km}}{\text{hr}} = \frac{-135}{\sqrt{117}} \frac{\text{km}}{\text{hr}} \approx -12.48 \frac{\text{km}}{\text{hr}}$