Strategy for Solving a Related Rates Problem

Example: The length of a rectangle is decreasing at the rate of 5 cm/s while the width is increasing at the rate of 3 cm/s. Find the rate of change of the area of the rectangle when $l = 10$ cm and $w = 8$ cm

Step 1: Write each rate in the problem (including the one you are trying to find) as a derivative.

$\frac{dl}{dt} = -5$ cm/s $\quad \frac{dw}{dt} = 3$ cm/s $\quad$ Find $\frac{dA}{dt}$

Step 2: Identify the dependent variables in the derivatives. Find an equation relating the dependent variables. You will often need a formula from geometry for this.

$$A = lw$$

Step 3: Differentiate both sides of the equation in step 2 with respect to the independent variable time. The result will be an equation relating the rates in the problem (relating the derivatives)

$$\frac{dA}{dt} = \frac{d}{dt}(l \cdot w)$$

$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$

Step 4: Substitute values given in the problem for the derivatives and other variables in the equation and solve the equation for the requested derivative.

$$\frac{dA}{dt} = 10\text{cm}\left(3\frac{\text{cm}}{\text{s}}\right) + 8\text{cm}\left(-5\frac{\text{cm}}{\text{s}}\right)$$

$$\frac{dA}{dt} = -10\frac{\text{cm}^2}{\text{s}}$$

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Example: The length of a rectangle is decreasing at the rate of 5 cm/s while the width is increasing at the rate of 3 cm/s. Find the rate of change of the area of the rectangle when $\ell = 10$ cm and $w = 8$ cm.

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$\frac{d\ell}{dt} = -5$ cm/s $\qquad \frac{dw}{dt} = 3$ cm/s $\qquad$ Find $\frac{dA}{dt}$

Step 2: Identify the dependent variables in the derivatives. Find an equation relating the dependent variables. You will often need a formula from geometry for this.

$$A = \ell w$$

Step 3: Differentiate both sides of the equation in step 2 with respect to the independent variable time. The result will be an equation relating the rates in the problem (relating the derivatives)

$$\frac{dA}{dt} = \frac{d}{dt}(\ell \cdot w)$$

$$\frac{dA}{dt} = \ell\frac{dw}{dt} + w\frac{d\ell}{dt}$$

Step 4: Substitute values given in the problem for the derivatives and other variables in the equation and solve the equation for the requested derivative.

$$\frac{dA}{dt} = 10\text{cm}\left(3\frac{\text{cm}}{\text{s}}\right) + 8\text{cm}\left(-5\frac{\text{cm}}{\text{s}}\right)$$

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