Strategy for Solving Related Rates Problems (Part 2)

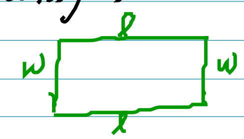
Example: The length of a rectangle is decreasing at the rate of 5 cm/s while the width is increasing at the rate of 3 cm/s. Find the rate of change of the perimeter of the rectangle when $l = 10$ cm and $w = 8$ cm

Step 1: Write each rate in the problem as a derivative.

$\frac{dl}{dt} = -5 \frac{cm}{s}$ $\quad \frac{dw}{dt} = +3 \frac{cm}{s}$ $\quad$ Find $\frac{dP}{dt}$

Step 2: Identify the dependent variables in the derivatives. Find an equation relating the dependent variables.

$$P = 2l + 2w$$



Step 3: Differentiate both sides of the equation in step 2 with respect to the independent variable t. The result will be an equation relating the rates in the problem (relating the derivatives.)

$$P = 2l + 2w$$

$$\frac{d}{dt}(P) = \frac{d}{dt}(2l + 2w)$$

$$\Longrightarrow \frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$$

Step 4: Substitute values and solve the equation from step 3 for the requested derivative.

$$\frac{dP}{dt} = 2\left(-5\frac{cm}{s}\right) + 2\left(3\frac{cm}{s}\right)$$

$$\frac{dP}{dt} = -4\ \frac{cm}{s}$$

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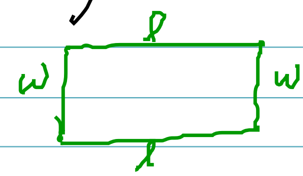
Example: The length of a rectangle is decreasing at the rate of 5cm/s while the width is increasing at the rate of 3 cm/s. Find the rate of change of the perimeter of the rectangle when $l = 10$ cm and $w = 8$ cm

Step 1: Write each rate in the problem as a derivative.

$$\frac{dl}{dt} = -5\,\frac{cm}{s} \qquad \frac{dw}{dt} = +3\,\frac{cm}{s} \qquad \text{Find } \frac{dP}{dt}$$

Step 2: Identify the dependent variables in the derivatives. Find an equation relating the dependent variables.

$$P = 2l + 2w$$



Step 3: Differentiate both sides of the equation in step 2 with respect to the independent variable t. The result will be an equation relating the rates in the problem (relating the derivatives.)

$$P = 2l + 2w$$

$$\frac{d}{dt}[P] = \frac{d}{dt}(2l + 2w)$$

$$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$$

Step 4: Substitute values and solve the equation from step 3 for the requested derivative.

$$\frac{dP}{dt} = 2\left(-5\,\frac{cm}{s}\right) + 2\left(3\,\frac{cm}{s}\right)$$

$$\frac{dP}{dt} = -4\,\frac{cm}{s}$$