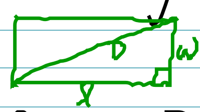
Strategy for Solving Related Rates Problems (Part 3)

Example: The length of a rectangle is decreasing at a rate of 5 cm/s, while the width is increasing at a rate of 3 cm/s. Find the rate of change of the diagonal with respect to time when $l = 10$ cm and $w = 8$ cm.

Step 1: Write each rate in the problem as a derivative

$\frac{dl}{dt} = -\frac{5\text{ cm}}{s}$ $\qquad \frac{dw}{dt} = +\frac{3\text{ cm}}{s}$ $\qquad$ Find $\frac{dD}{dt}$

Step 2: Identify the dependent variables from the derivative notation in Step 1. Find an equation relating the dependent variables.



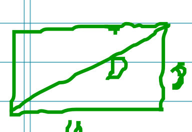
$D^2 = l^2 + w^2$

Step 3: Differentiate both sides of the equation from Step 2 with respect to the independent variable t. The result will be an equation relating the rates (derivatives) in the problem.

$$D^2 = l^2 + w^2$$
$$\frac{d}{dt}(D^2) = \frac{d}{dt}(l^2 + w^2)$$
$$\longrightarrow \quad 2D\frac{dD}{dt} = 2l\frac{dl}{dt} + 2w\frac{dw}{dt}$$
$$D\frac{dD}{dt} = l\frac{dl}{dt} + w\frac{dw}{dt}$$

Step 4: Substitute values and solve the equation from step 3 for the requested derivative (rate.)

$$\longrightarrow D\frac{dD}{dt} = l\frac{dl}{dt} + w\frac{dw}{dt}$$



$$(\sqrt{164}\text{ cm})\frac{dD}{dt} = (10\text{ cm})\left(\frac{-5\text{ cm}}{s}\right) + (8\text{ cm})\left(\frac{3\text{ cm}}{s}\right)$$

$D^2 = 100 + 64 \Rightarrow$
$D = \sqrt{164}$ cm

$$\frac{dD}{dt} = \frac{-26\frac{\text{cm}^2}{s}}{\sqrt{164}\text{ cm}} = \frac{-26}{\sqrt{164}}\frac{\text{cm}}{s} = \frac{-26}{\sqrt{4}\sqrt{41}}\frac{\text{cm}}{s}$$
$$= \frac{-13}{\sqrt{41}}\frac{\text{cm}}{s}$$

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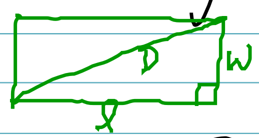

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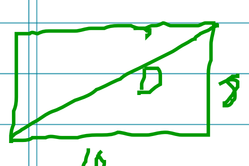

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