Strategy for Solving Related Rates Problems (Part 4)

The length of a rectangle is decreasing at a rate of 5 cm/s, while the area increases at a rate of 2 cm²/s. Find the rate at which the width is changing when $w = 8$ cm and $A = 12$ cm².

$\frac{dl}{dt} = -5$ cm/s $\frac{dA}{dt} = +2$ cm²/s Find $\frac{dw}{dt}$

$A = l \cdot w$

$\frac{dA}{dt} = \frac{d}{dt}(l \cdot w)$

$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

$2\frac{cm^2}{s} = \left(\frac{3}{2}cm\right)\frac{dw}{dt} + 8cm\left(-5\frac{cm}{s}\right)$

$2\frac{cm^2}{s} = \left(\frac{3}{2}cm\right)\frac{dw}{dt} - 40\frac{cm^2}{s}$

$2\frac{cm^2}{s} + 40\frac{cm^2}{s} = \left(\frac{3}{2}cm\right)\frac{dw}{dt}$

$42\frac{cm^2}{s} = \frac{3}{2}cm\,\frac{dw}{dt}$

$\frac{42\frac{cm^2}{s}}{\frac{3}{2}cm} = \frac{dw}{dt}$

$42\frac{cm^2}{s} \cdot \frac{2}{3cm} = \frac{dw}{dt}$

$\frac{dw}{dt} = \frac{84}{3}\frac{cm}{s}$

Finding l

$A = 12\,cm^2$, $8cm = w$

l

$l \cdot w = A$

$l \cdot 8cm = 12cm^2$

$l = \frac{12cm^2}{8cm}$

$l = \frac{3}{2}cm$

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$\longrightarrow \frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$ ←

$\longrightarrow 2\frac{cm^2}{s} = \left(\frac{3}{2}cm\right)\frac{dw}{dt} + 8cm\left(-5\frac{cm}{s}\right)$

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