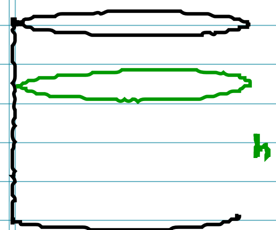
Strategy for Solving Related Rates Problems (Part 5)

Water is being pumped out of a cylindrical tank at a rate of 10 ft³/min. The tank is a right circular cylinder and has a radius of 2 ft and a height of 5 ft. How fast is the water level falling when the water level in the tank is 4 ft?



Step 1: Write each rate in the problem as a derivative.

$\frac{dV}{dt} = -10\ ft^3/min$ Find $\frac{dh}{dt}$

Step 2: Identify the dependent variables from the derivative notation in Step 1. Find an equation relating them. $V = \pi r^2 h$

Step 3: Differentiate both sides of the equation from step 2 with respect to the independent variable t. The result will be an equation relating the rates (derivatives) in the problem

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left(r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt}\right)$

$\frac{dV}{dt} = \pi\left(r^2 \frac{dh}{dt} + 0\right)$

$\frac{dV}{dt} = \frac{d}{dt}(\pi r^2 h) \Rightarrow \boxed{\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}}$

Step 4: Substitute values and solve the equation from Step 3 for the requested rate (derivative)

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$-10\ \frac{ft^3}{min} = \pi (2ft)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-10}{4\pi} \frac{\frac{ft^3}{min}}{ft^2} = \frac{-5}{2\pi} \frac{ft}{min}$$

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