Solutions to Introducing Continuity

Problem 0) For the position function for our car scenario (see function and graph below), evaluate the items in parts a), b), c) and d).

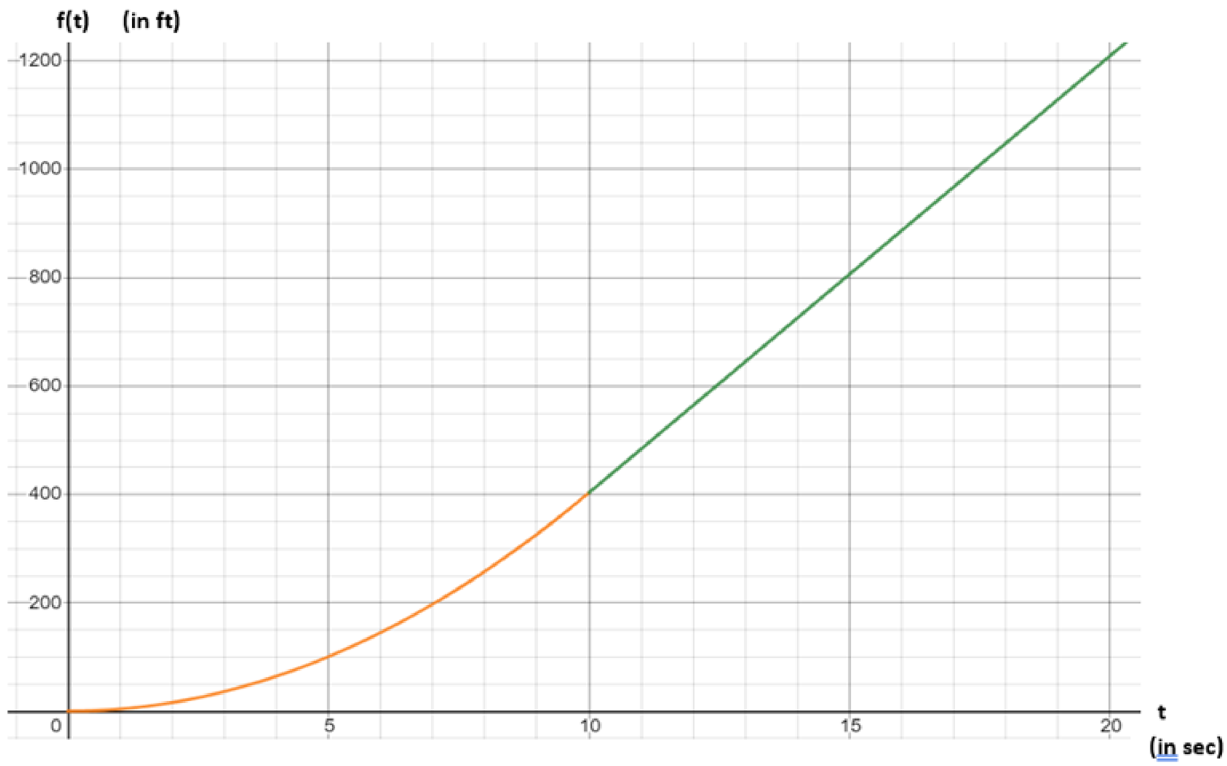
a) s(10) = $\left(\frac{121}{30}\right) \cdot 10^2 = \frac{1210}{3} = 403.\overline{3}$

b) $\lim_{t\to 10^-} s(t) = 403.\overline{3}$

c) $\lim_{t\to 10^+} s(t) = 403.\overline{3}$

d) $\lim_{t\to 10} s(t) = 403.\overline{3}$

$$s(t) = \begin{cases} \left(\frac{121}{30}\right)t^2 & for\ 0 \le t \le 10 \\ \left(\frac{1}{3}\right)(242t - 1210) & for\ t > 10 \end{cases}$$ where t is in sec. and s(t) is in ft.



Problem 1) For the function in Example 1 (see function and graph below), evaluate the items in parts a), b), c) and d).

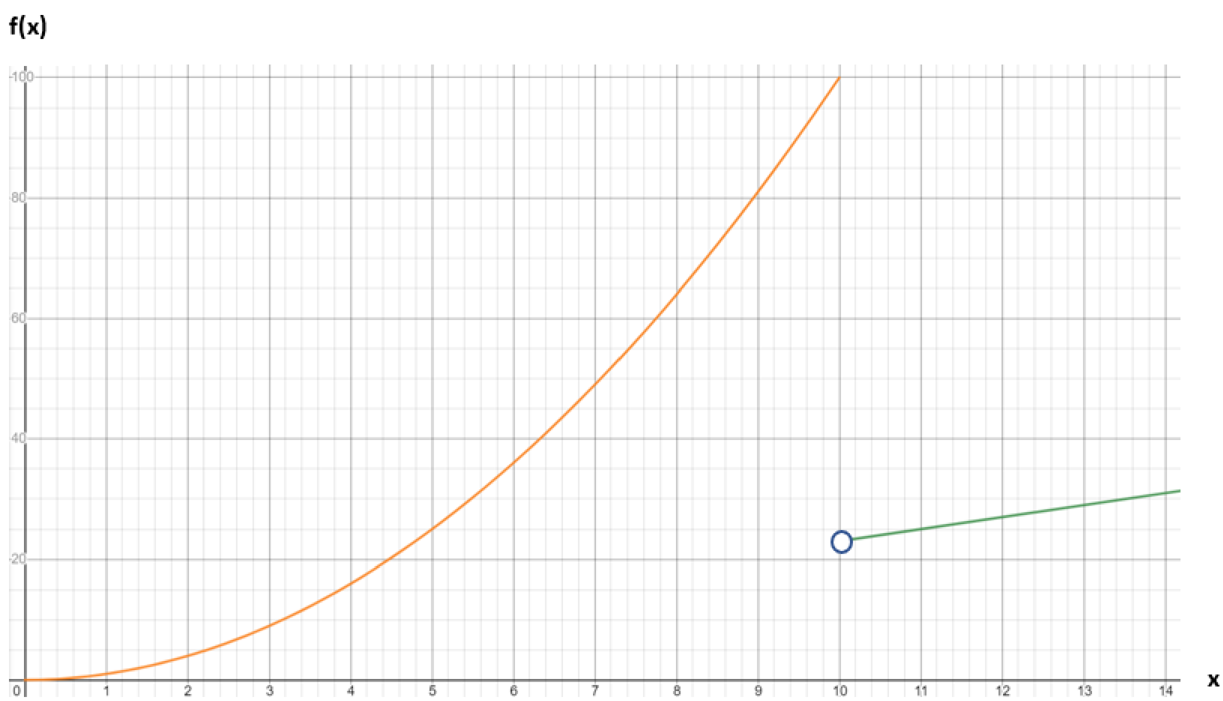
a) f(10) = $10^2 = 100$

b) $\lim_{x \to 10^-} f(x) = 100$

c) $\lim_{x \to 10^+} f(x) = 23$

d) $\lim_{x \to 10} f(x)$ *does not exist (DNE)*

$$f(x) = \begin{cases} x^2 & \text{for } 0 \le x \le 10 \\ 2x + 3 & \text{for } x > 10 \end{cases}$$



Problem 2) For the function in Example 2 (see function and graph below), evaluate the items in parts a), b), c) and d).

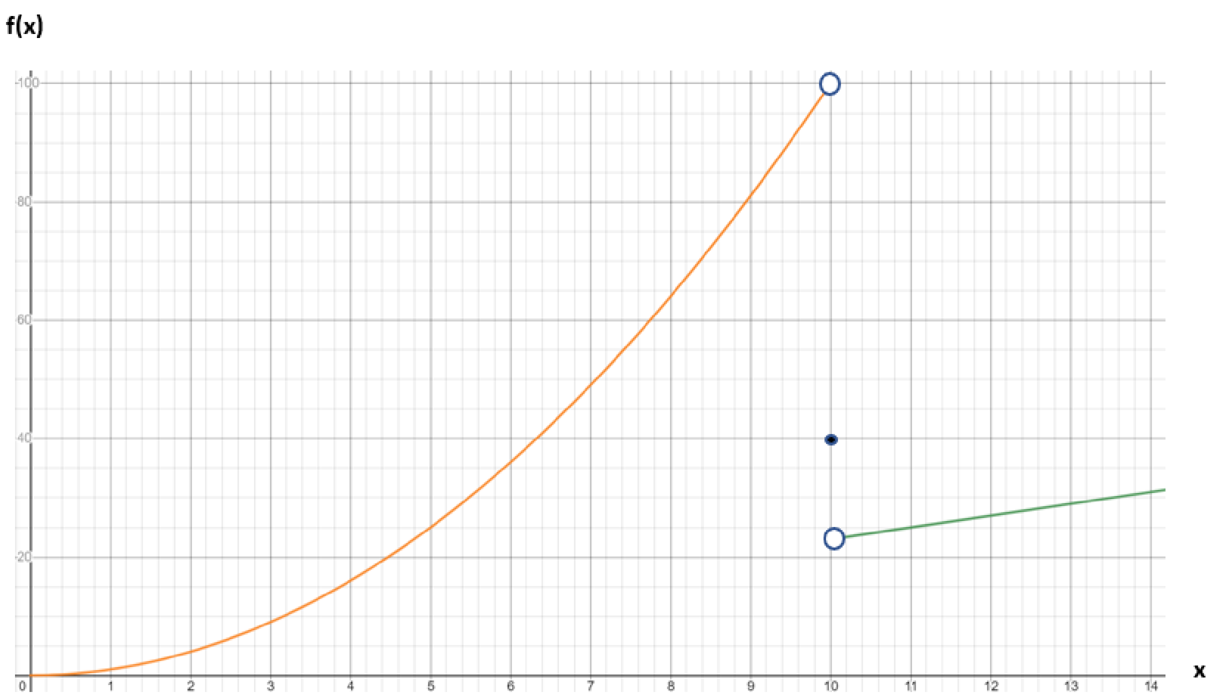
a) f(10) $= 40$

b) $\lim_{x\to 10^-} f(x) = 100$

c) $\lim_{x\to 10^+} f(x) = 23$

d) $\lim_{x\to 10} f(x)\ DNE$

$$f(x) = \begin{cases} x^2 & for\ 0 \le x < 10 \\ 40 & for\ x = 10 \\ 2x+3 & for\ x > 10 \end{cases}$$



<u>Problem 3)</u> For the function in Example 3 (see function and graph below), evaluate the items in parts a), b), c) and d).

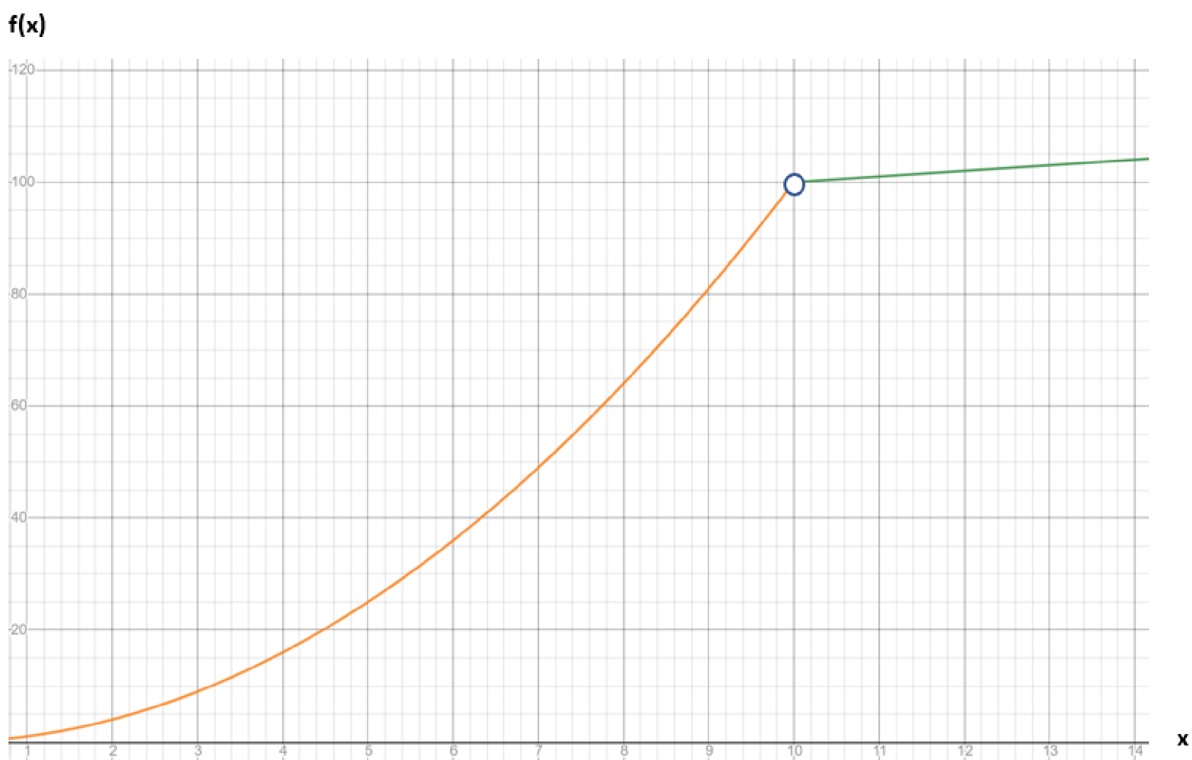
a) f(10) DNE

b) $\lim_{x \to 10^-} f(x) = 100$

c) $\lim_{x \to 10^+} f(x) = 100$

d) $\lim_{x \to 10} f(x) = 100$

$$f(x) = \begin{cases} x^2 & for\ 0 \leq x < 10 \\ 90 + x & x > 10 \end{cases}$$



Problem 4) For the function in Example 4 (see function and graph below), evaluate the items in parts a), b), c) and d).

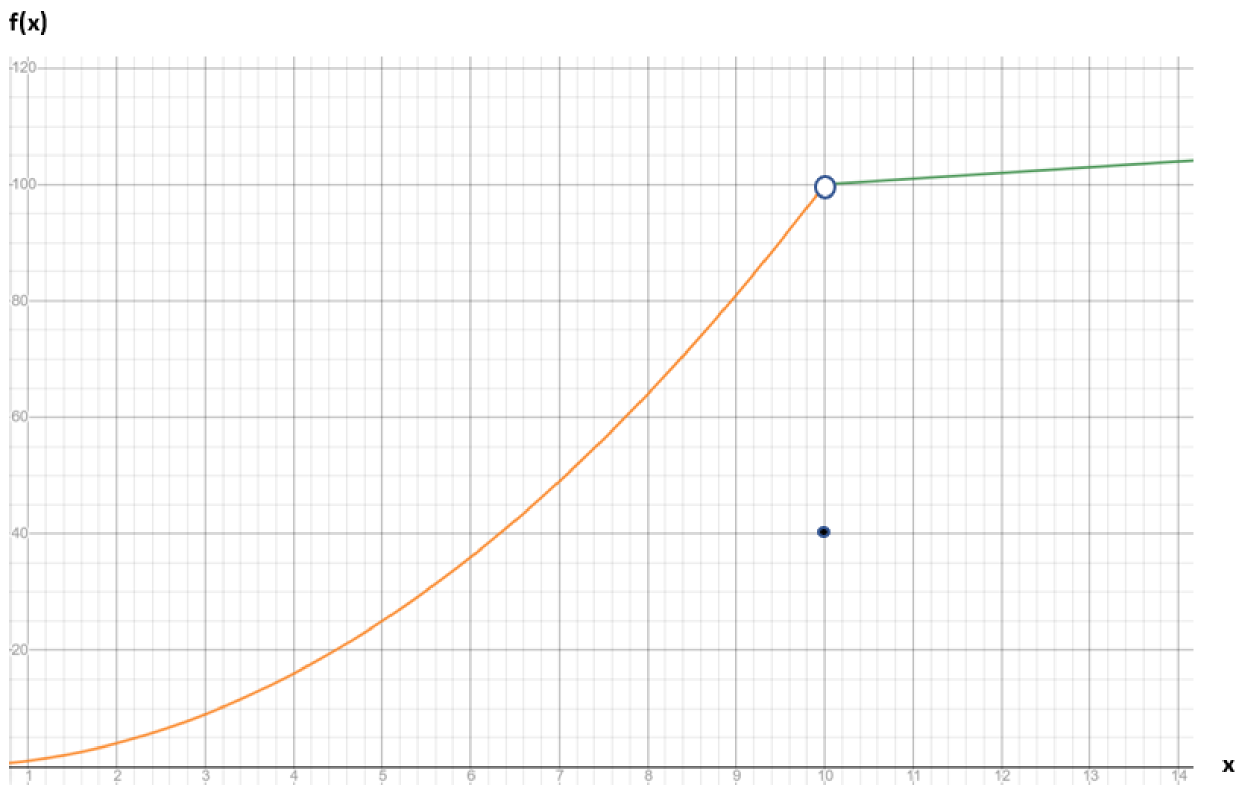
a) f(10) $= 40$

b) $\lim_{x\to 10^-} f(x) = 100$

c) $\lim_{x\to 10^+} f(x) = 100$

d) $\lim_{x\to 10} f(x) = 100$

$$f(x) = \begin{cases} x^2 & for\ 0 \le x < 10 \\ 40 & for\ x = 10 \\ 90 + x & for\ x > 10 \end{cases}$$



The function s in the car scenario (see Problem 0) is continuous at $t = 10$ while the functions f in Problems 1,2,3,4 are not continuous at $x = 10$. We say the functions in Problems 1,2,3,4 are discontinuous at $x = 10$.

For a function f to be continuous at $x = 10$, three conditions must be met. Based on the Problems 0 – 4, and the results of questions a) – d), what do you think these three conditions are?

i) $f(10)$ must exist

ii) $\lim_{x\to 10} f(x)$ must exist

iii) $f(10)$ and $\lim_{x\to 10} f(x)$ must be equal

Explain what these conditions mean in terms of the car scenario.

In terms of the car scenario, s(10) exists means the car had a position at 10 seconds, in other words, the car was on the road at t =10 seconds. The fact that $s(10) = 403.\overline{3}$ means that $403.\overline{3}$ feet was the position of the car on the road when t was 10 seconds.

$\lim_{t\to 10} s(t) = 403.\overline{3}$ means that when the time is close to 10 seconds, the position of the car is close to $403.\overline{3}\,ft$. More precisely, the position of the car can be found to be as close to $403.\overline{3}\,ft$ as you please, just by choosing a time sufficiently close to 10 seconds.

$s(10)$ and $\lim_{t\to 10} s(t)$ being equal means that when the time is close to 10 seconds the position of the car is close to the position of the car at 10 seconds.

Why must the position function for the car be continuous at $t = 10$?

The position function of the car not being continuous at t = 10 would require a physical impossibility, like a Star Trek transporter making the car at 10 seconds suddenly disappear and then reappear afterwards down the road.

Which of the conditions fails for each of the functions in problems 1-4?

Item ii) fails for Problems 1 and 2 producing, what we call a JUMP discontinuity in each problem.

Item i) fails for Problem 3, producing what we call a REMOVABLE discontinuity. (If defined f(10) to be 100, then the function would be continuous and the discontinuity would be removed.)

Item iii) fails for Problem 4, producing a REMOVABLE discontinuity.

Generalize your previous result. For a function to be continuous at $x = a$, what three conditions must be met?

i) $f(a)$ must exist

ii) $\lim_{x \to a} f(x)$ must exist

iii) $f(a)$ and $\lim_{x \to a} f(x)$ must be equal

This is the textbook definition of a function being continuous at a point.