

## Related Rates Overview: Strategy for Solving Related Rates Problems

In a typical related rates problem, you will be given some rates and asked to find a rate. These rates are all instantaneous rates. The independent variable is always time. The rates that you are given will be constant (that is, the same for all times.) However the rate that you are trying to find often is not constant (differs for various times.)

Related rates problems can be solved in the following way:

Step 1: Write each rate in the problem (including the one you are trying to find) as a derivative.

Step 2: Identify the dependent variables in the derivatives. Find an equation relating the dependent variables. You will often need a formula from geometry for this.

Step 3: Differentiate both sides of the equation from Step 2 with respect to the independent variable, time. The result will be an equation relating the rates in the problem (relating the derivatives).

Step 4: Substitute the values that you are given in the problem for the derivatives and the variables and solve for the derivative for which the question is asking.

**Common mistake:** substituting values (step 4) before step 3.

## Teaching Notes

- 1) Many times through the years I have heard students say that they don't know how to start a particular related rates problem. So I began promoting the use of this four-step process to help scaffold learning. I would argue that the primary purpose of having students solve related rates problems is not to have them engage in problem solving, but rather that they better understand the mathematics related to derivatives as instantaneous rates of change. By using this process, students can gain confidence in solving related rates problems as they apply the process to problems in various contexts, and can focus upon the mathematics related to derivatives. Included in these OER resources are seven pencasts, each illustrating the use of the process on a particular related rates problem. The problems in the pencasts have been varied to provide an opportunity for a student using this resource to be exposed to a variety of issues regarding derivatives and rates which often trouble students. The fact that the contexts of these examples are limited allows students to have the opportunity to gain independent experience solving related rates problems in new contexts which you will likely want to provide them in problem sets.
- 2) I like to begin a discussion of related rates by comparing related rates problems (as described at the beginning of this handout) with problems in which a function  $y = f(x)$  is provided, and one is asked to find the instantaneous rate of change of  $y$  with respect to  $x$  at a particular value of  $x$ , something which students typically have done prior to being exposed to related rates problems.