Employment, Wage, and Output Dynamics under External Habit Formation and Efficiency Wages

Matthew L. Booth*
Middle Tennessee State University
Murfreesboro TN 37132
mbooth@mtsu.edu

February 9, 2018

Abstract

I construct an equilibrium model that combines external habit formation in consumption and efficiency wages arising from imperfectly observable effort to evaluate wage, employment, and output dynamics under fiscal and technology shocks. At certain levels of insurance and habit formation employment-output correlations and output volatilities match US data better than a model without habit formation. However, increased employment volatility and counterfactual negative wage-employment correlations emerge. I use impulse response functions to explain the mechanisms that give rise to the observed changes in second moments.

Keywords: Unemployment, Habit formation, Partial Insurance
JEL Classification: E24, E21

*I am grateful to Gregory Givens for useful discussions of the challenges of modeling wage dispersion arising from heterogeneous work histories. Thanks to Stuart Fowler for advice on research motivation and editorial comments.
1 Introduction

This study analyzes the dynamics of a shirking efficiency wage model along the lines of Alexopoulos (2004). The key addition to the model is habit formation in consumption. I report simulation results that are directly comparable to Alexopoulos’s partial insurance and full insurance setups. Indeed, with certain parameter values the model can be made equivalent to either case. In my model I parameterize the insurance level in a manner similar to that of Givens (2008). For certain values of the habit formation and insurance parameters, output volatility and output-employment correlations match the U.S. data better than those produced by the Alexopoulos (2004) partial insurance model, although increased employment and counterfactual negative employment-wage correlations emerge.

A model incorporating both an efficiency wage and habit formation allows for a discussion of the dynamics arising from habit formation in an environment with structural unemployment. I use the model to focus on output-employment correlations and output volatility, observing the effects of habit formation under different levels of insurance. The insurance level models the extent to which workers contribute to a fund for the purpose of augmenting the income of the unemployed. It affects the penalty for shirking, and thus wage dynamics. With the addition of habit formation, dynamics are also affected through the utility of consumption, altering shirking penalty effects. While it is known that habit formation creates delayed and smoothed responses to shocks, the dynamics that result from including both habit formation and shirking efficiency wages with partial insurance are not well understood.¹

The effects of habit formation, when combined with a partial-insurance shirking efficiency wage, are qualitatively more complex than a simple smoothing and delaying of responses. Alexopoulos (2004) finds that the shirking penalty effects of technology and fiscal shocks allow for improved wage and employment volatilities and correlations. However, her partial insurance model, which can be taken as a special case of mine, also produces excessive output volatility and output-employment correlations. In terms of a comparison to Alexopoulos’s results, my model generates an improvement in output and employment dynamics that comes at the cost of a counterfactual negative wage-employment correlation and increased volatility of both wages and employment. Due to opposed effects of partial insurance and habit formation, the improvements can be achieved at different parameter combinations that imply different ratios of employed to unemployed consumption. Rather than calibrate to such a consumption ratio as Alexopoulos (2004) does, I report its implied steady-state value for different levels of habit and insurance. Thus, it is shown that while habit and insurance values that generate the improved results are not unique, each pair of values implies a distinct consumption ratio.

First, I present the model with habit formation and an exogenous insurance level. I use external habit formation for this formulation, so workers are homogeneous with respect to the effects of habit formation. The worker’s decision whether to shirk leads to an efficiency wage and steady-state unemployment, exactly as described by Alexopoulos (2004). Insurance and habit formation levels affect shirking decisions, and thus the wage, in addition to affecting

---

2 Insurance level is parameterized in a way similar to that of Givens (2008) but the insurance parameter does not have a direct interpretation as a consumption ratio unless habit formation is zero.

3 Because workers have different previous individual consumption levels due to previous employment status, allowing different reference levels (internal habit formation) would result in different IC constraints and wages.
investment and consumption. Next, I focus on the short-run dynamics of employment, output, and wages, in simulations with technology and fiscal shocks. I solve a linearized system in log deviations from the steady states, and use the solution to simulate an economy. I describe the results of simulations, discussing the effects of habit formation and insurance levels on output volatility and output-employment correlation. Effects that depend on both the insurance and habit formation levels interact to bring about the improvements. I use impulse response diagrams to aid the discussion. The tables and figures I provide can be compared directly to Alexopoulos’s (2004) findings.

2 The Model

Firms have imperfect capability to monitor worker effort, and exerting effort causes disutility for workers. Workers decide whether to shirk. Firms choose the smallest wage such that the expected utility of shirking detection equals the utility from exerting effort. Several things can vary in this model, including the probability of detection and the nature and extent of the penalty for shirking.\(^4\) The extent to which workers share unemployment risk by redistributing consumption alters the incentive compatibility (IC hereafter) constraint that governs the wage decision of the firm.\(^5\) My model adds habit formation to the utility function. Where Alexopoulos (2004) considers full and partial insurance setups, with partial insurance defined as the level where consumption is the same for detected shirkers and the unemployed, I consider different insurance arrangements by adding an exogenous parameter


\(^5\)Insurance options ranging from zero to perfect have been considered. Alexopoulos shows full insurance is equivalent to the model of Hansen (1984) and Givens (2008) allows the insurance level to vary in the full range.
for insurance analogous to that used by Givens (2008).

A representative family rents capital to a representative firm. The family make an investment decision and allocates consumption among the members, attempting to maximize the expected utility of its members. The firm chooses employment and wage levels with the objective of maximum profit, knowing a shirking worker produces no output. Employed members decide whether to shirk. Since shirkers do no work, the firm chooses a wage level to just ensure that shirking does not occur.\(^6\) In other words, where a worker is indifferent between shirking and working.\(^7\)

Government expenditures \(G_t\) appears as a cost (tax) to the family, and is added to simulate fiscal shocks. The technology coefficient in the production function \(A_t\) is used to simulate technology shocks. These are modeled with the AR processes

\[ \tilde{G}_t = \rho_A \tilde{G}_{t-1} + \epsilon_{G_t}, \quad \text{and} \]

\[ \tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A_t}, \quad \text{(2)} \]

where \(\epsilon_{A_t}\) and \(\epsilon_{G_t}\) are serially uncorrelated innovations with mean zero and standard deviations \(\sigma_a\) and \(\sigma_g\).\(^8\) I use parameter values from Alexopoulos's (2004) GMM estimation.\(^9\)

---

\(^6\)This is a result of the worker utility maximization problem described in detail by Alexopoulos (2004).

\(^7\)I derive the function for effort in terms of wage from the indifference condition in Appendix A.

\(^8\)The equations presented here are expressed as log deviations from mean to equate to the linearized system presented in the solution section, while Alexopoulos (2004) writes them in levels. The two are exactly equivalent (\(\rho\) values are the same.)

\(^9\)My simulation differs from hers in that I do not include technology and government spending growth. My technology and government spending shocks are independent of one-another. The results I am interested in are not affected.
2.1 Family

I add habit formation to utility using a reference consumption level determined by the consumption levels of unemployed and employed members last period. The reference level is

$$\bar{c}_t = N_t c^e_t + (1 - N_t) c^u_t,$$

(3)

where $N_t \in [0, 1]$ is the employment level, and $c^e_t$ and $c^u_t$ are the consumption levels of employed and unemployed family members, respectively. Utility for an employed worker, accounting for habit formation, is

$$U_t(c^e_t, \bar{c}_{t-1}) = \ln (c^e_t - b\bar{c}_{t-1}) + \theta \ln (T - he_t - \zeta),$$

(4)

where $b$ is the habit formation parameter, $T$ is the time endowment of an individual, $e_t$ is the effort level expended, $h$ is work hours, and $\zeta$ is the fixed cost of exerting any effort greater than zero. An unemployed worker experiences utility based on unemployed consumption, and no disutility from working,

$$U_t(c^u_t, \bar{c}_{t-1}) = \ln (c^u_t - b\bar{c}_{t-1}) + \theta \ln (T).$$

(5)

A detected shirking employed worker consumes at the shirking consumption level, $c^s_t$, and experiences utility\textsuperscript{10}

$$U_t(c^s_t, \bar{c}_{t-1}) = \ln (c^s_t - b\bar{c}_{t-1}) + \theta \ln (T).$$

(6)

\textsuperscript{10}Since the IC constraint on wage assures employers will choose a wage which prevents all shirking, this utility expression applies to no one.
The family decides a consumption contribution $c_i^f$ to be given to each member. The shirking penalty $s \in [0, 1]$ is the fraction of the full wage that a detected shirker will receive. The penalty for detected shirking, then, is $w_i(1 - s)$. Each employed family member contributes an amount $F_t$ as insurance, to be paid to the unemployed. Thus, consumption levels are

$$c^u_t = c^f_t + \frac{N_t}{1 - N_t} F_t, \quad (7)$$

for the unemployed,

$$c^s_t = c^f_t + s w_i h - F_t, \quad (8)$$

for a detected shirker, and

$$c^e_t = c^f_t + w_i h - F_t, \quad (9)$$

for the employed. The insurance contribution from each member is exogenous to the family, determined by the wage chosen by the representative firm, and $\sigma$, the insurance parameter, by the rule

$$F_t = \sigma (1 - N_t) hw_t. \quad (10)$$

The family also chooses an investment level, and thus a capital level, $K_{t+1}$, for next period, subject to a budget constraint including investment in new capital and taxes.\footnote{Because tax paid is equal to $G_t$, $G_t$ is substituted for tax cost in the budget constraint presented here.} The
family makes its choices to solve

$$\max_{\{c_t, K_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ (N_t) \ln \left( \frac{c^e_t - b\bar{c}_{t-1}}{c^e_{t+1} - b\bar{c}_{t-1}} \right) + (N_t) \theta \ln \left( T - h c_t - \zeta \right) \right\} \right\} , \quad (11)$$

subject to $\bar{c}_t \leq [r_t K_t - G_t - [K_{t+1} - (1 - \delta) K_t]] , \quad (12)$

where $\delta \in [0, 1]$ is the depreciation rate and $r_t$ is the rental rate of capital. Maximizing the Lagrangian, the first order conditions are

$$\frac{(N_t)}{(c^e_t - b\bar{c}_{t-1})} + \beta \frac{(N_{t+1})(-b)}{(c^e_{t+1} - b\bar{c}_{t-1})} + \frac{(1 - N_t)}{(c^u_t - b\bar{c}_{t-1})} + \beta \frac{(1 - N_{t+1})(-b)}{(c^u_{t+1} - b\bar{c}_{t})} = \lambda_t , \quad (13)$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) , \quad (14)$$

where $\lambda_t$ is the Lagrangian multiplier associated with Equation (12).

### 2.2 Member Incentive Compatibility Constraint

The IC (incentive compatibility) constraint will apply to members who are employed. The IC constraint comes from calculating the wage level which will make a worker indifferent between shirking and working. The effort level is a function of the wage. Defining $\chi_t$ as

$$\chi_t = \frac{c^e_t - b\bar{c}_{t-1}}{c^u_t - b\bar{c}_{t-1}} , \quad (15)$$
we can show $\chi_t$ is constant, determined by the relationship to time endowment, fixed effort cost, utility weight of leisure, detection probability, and shirking penalty defined by

$$\left((T - \zeta) \chi^{1+\frac{d}{\theta}} - T \chi\right) (1 - s) = T \left(\frac{d}{\theta}\right) (1 - s \chi)(\chi - 1).$$

(16)

The IC constraint arising from the worker’s shirking decision is obtained by finding the effort level which makes the worker indifferent between shirking and working at a given wage. The effort level is shown to be constant, because $\chi_t$ is constant. The effort function is used as a constraint in the firm’s problem,

$$E_t = \Delta(w_t) = -\frac{T}{h} \left(\chi_t\right)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}. \tag{17}$$

It follows directly from consumption definitions that

$$\left(1 - s\right)\frac{\chi}{\chi - 1} h w_t = c_t - b c_{t-1} \tag{18}$$

and $\mu(\sigma) = 1/\chi$, a function of the insurance parameter $\sigma$, following

$$\frac{c_t^\mu - b c_{t-1}}{c_t^e - b c_{t-1}} = \mu(\sigma) = 1 - \frac{1 - \sigma}{1 - s} \left(\frac{\chi_t - 1}{\chi_t}\right). \tag{19}$$

My definition of $\chi$ differs from that of Alexopoulos (2004) when habit formation is not zero.

---

12 Appendix D Uses the wage FOC from the firm’s problem (Solow condition) and the consumption equations to demonstrate this.

13 Appendix A.

14 Appendix B.

15 Appendix B. It is still true that the partial insurance case, as defined by Alexopoulos (2004), comes about when $s=\sigma$, as Givens (2008) shows when he introduces the insurance level parameter $\sigma$. 

9
While $\chi$ is constant, $c_t^a/c_t^e$ is not, but I report its steady-state value in my results, for a measure of insurance level which can be compared to other results, or potentially calibrated to observed data, such as the income ratio estimated by Gruber (1997).\(^{16}\) When $b=0$, of course, these variables reduce to the values seen elsewhere and become constant.

### 2.3 Firm

The firm chooses a wage, an employment level, an effort level, and capital, to maximize profit. Costs are the wage bill and capital rental. The production function,

$$Y_t = A_t K_t^\alpha ((N_t) h E_t)^{1-\alpha},$$  \hspace{1cm} (20)

combines effective labor and capital.\(^{17}\) The incentive compatibility constraint,

$$E_t = \Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h},$$  \hspace{1cm} (21)

enters the firm’s decision as a binding constraint on the wage. The smallest wage that will prevent shirking is chosen. The firm maximizes profit by solving

$$\max_{\{w_t, N_t, E_t, K_t\}} \left( A_t K_t^\alpha (h E_t N_t)^{1-\alpha} - w_t h N_t - r_t K_t \right).$$  \hspace{1cm} (22)

---

\(^{16}\)I do not calibrate to this. I recover it from the steady state which yields a certain employment level.

\(^{17}\)The production function incorporates the finding that no shirking happens in equilibrium, so there is only one employment level and it fully contributes to production.
Maximizing profit yields the first order conditions\textsuperscript{18},

\[ r_{t+1} = \frac{\alpha Y_{t+1}}{K_{t+1}}, \]  
(23)

\[(1 - \alpha)A_t K_t^\alpha (hE_t N_t)^{-\alpha} hE_t = w_t h, \text{ and} \]
(24)

\[ \frac{(w_t) \Delta'(w_t(e))}{\Delta(w_t(e))} = 1. \]  
(25)

\subsection{2.4 General equilibrium}

Expenditure equals income in equilibrium, and demand and supply for capital are equal. Aggregating the family budget constraint yields

\[ C_t + G_t + I_t = Y_t. \]  
(26)

\section{3 Calibration}

In each simulation I solve for parameters affected by changes in the utility function due to adding habit formation in order to yield a steady state employment rate of 0.941.\textsuperscript{19} Other calibrations are borrowed from Alexopoulos’s (2004) GMM estimates. Yet others are taken from other literature following Alexopoulos (2004). I depart from her estimates where necessary, since the utility function is altered under habit formation ($b>0$). $d/\theta$ is affected, taking a different value under different values of $b$, for instance. Since $(c_t^u - b\tilde{c}_{t-1}) / (c_t^u - b\tilde{c}_{t-1})$ is constant, $c_t^u / c_t^e$ is not constant. I solve for \[ \chi_t = (c_t^e - b\tilde{c}_{t-1}) / (c_t^u - b\tilde{c}_{t-1}) \] at each parameter.

\textsuperscript{18} Appendix C shows derivation of the Solow condition.

\textsuperscript{19} This is the average from U.S. historical data.
set considered, because it is a constant value. If desired, one could calibrate to a $c_t^u/c_t^e$ as an steady state, but in this study the dynamics of consumption are not of direct interest, so I calibrate to an employment steady state. I report $c_t^u/c_t^e$ in all tables, and the trends in $c_t^u/c_t^e$ with changes in $b$ and $\sigma$ are not surprising. My constant $\chi$ (constant for given $\sigma$ and $b$) does not indicate a simple consumption ratio under any degree of habit formation besides zero. I use this relationship,

$$\left((T - \zeta) \chi^{\frac{d}{\theta}} - T \chi\right)(1 - s) = T \left(\frac{d}{\theta}\right)(1 - s\chi)(\chi - 1), \tag{27}$$

to solve for $\chi$, $d/\theta$, and $s$ in order to yield steady state $N = 0.941$, at each set of $\sigma$ and $b$ parameters considered. The parameters that do not change with changes in $b$ and $\sigma$, and their values, are $\{\beta = 0.9796, \delta = 0.0203, \sigma_g = 0.0133, \sigma_a = 0.0074, \rho_g = 0.9797, \rho_a = 0.9699, \alpha = 0.4574, \zeta = 16, T = 1369, \log(g/y) = -1.6870\}$.

4 Solution

I am interested in the dynamics of $W$ (wage), $N$ (employment), and $Y$ (output), in a simulation using the estimated variances and persistence of the government spending and technology disturbances. The linearized system is solved, and this solution is used to generate simulations. The results are expressed in log deviations from the steady state.

I generate impulse response experiments to provide insight into mechanisms for changes in

---

20 Alexopoulos (2004) does so, using a ratio derived from estimates by Gruber (1995). Givens (2008) generalizes to account for different values of this ratio, which characterizes the insurance level, since such a ratio is not directly measurable.

21 Here I depart somewhat from Alexopoulos’s model. She incorporates technology growth and a related government spending growth, where I have made the two series stationary and uncorrelated. In terms of implementing Klein’s solution method: the autocorrelation matrix of the forcing factors vector is diagonal.
second moments at different insurance and habit formation levels. For reference, I present the linear system here. Steady states are recalculated at every set of \(\sigma\) (insurance) and \(b\) (habit formation) values, so the constants in these equations differ for different values, where the steady states appear in the equations. Klein’s (2000) method yields a state-space solution in the predetermined and non-predetermined variable vectors. \(G_t\) and \(A_t\) are the exogenous forcing factors.

Defining \(\tilde{x}_t = \ln x_t - \ln x\), and using no subscript to indicate a steady-state value, the linearized system contains the equations

\[
\tilde{A}_t + \alpha K_t + (1 - \alpha)e\tilde{N}_t = \tilde{Y}_t, \tag{28}
\]

\[
\tilde{c}\tilde{c}_t + G\tilde{G}_t + K\tilde{K}_{t+1} - (1 - \delta)K\tilde{K}_t = Y\tilde{Y}_t, \tag{29}
\]

\[
\left(\frac{N}{e^\varepsilon - b\tilde{c}} - \frac{N}{e^u - b\tilde{c}}\right)\tilde{N}_t = \frac{N}{(e^\varepsilon - b\tilde{c})}\tilde{c}_t^e + \left(\frac{1-N}{e^u - b\tilde{c}}\right)\tilde{c}_t^u + \left(b\frac{N}{(e^\varepsilon - b\tilde{c})} + b\frac{1-N}{(e^u - b\tilde{c})}\right)\tilde{c}_{t-1} = \lambda\tilde{\lambda}_t, \tag{30}
\]

\[
\tilde{\lambda}_t = \beta \left(\frac{Y}{K} + 1 - \delta\right)\tilde{\lambda}_{t+1} + \beta\alpha\frac{Y}{K}\tilde{Y}_{t+1} - \beta\alpha\frac{Y}{K}\tilde{K}_{t+1}, \tag{31}
\]

\[
\tilde{Y}_t - \tilde{N}_t = \tilde{w}_t, \tag{32}
\]

\[
(1-s)\frac{\chi}{\chi - 1}hw\tilde{w}_t = c^e\tilde{c}_t^e - b\tilde{c}_t - 1, \tag{33}
\]

\[
c^u\tilde{c}_t^u - bc\tilde{c}_t - 1 = \left(1 - \frac{1 - \sigma}{1 - s}\right)\left(\frac{\chi}{\chi - 1}\right)\left(c^e\tilde{c}_t^e - b\tilde{c}_t - 1\right), \tag{34}
\]

\[
(Nc^e - Ne^u)\tilde{N}_t + Nc^e\tilde{c}_t^e + (1 - N)c^u\tilde{c}_t^u = \tilde{c}_t. \tag{35}
\]

\(^{22}\)The \(\sigma\) and \(\rho\) values are taken from Alexopoulos (2004). See the calibration section.
5 Results

Second moments are taken from repeated simulations, with the series HP-filtered ($\lambda = 1600$). I also supply impulse responses to aid the discussion of mechanisms for the second moment variation observed with different levels of habit formation and insurance. I report $c^u_t/c^e_t$ values recovered from the steady state consumption values, to see what the implied insurance level is in those terms.\(^{23}\)

My model is equivalent to Alexopoulos (2004) when my habit formation level is zero and insurance levels are set to certain levels.\(^{24}\) At certain $\sigma$ and $b$ values, the initial technology-shock effect on $N$ and the smoothed and delayed responses of all variables to shocks interact to yield lower $N-Y$ correlations and lower $Y$ volatility. This better matches U.S. Data than the model without habit formation. However, a much greater negative correlation between the wage and employment and increased wage and employment volatility are present, as compared to a model without habit formation. The improved second moments match the data most closely at sets of values ($\sigma$ and $b$) which are not unique. That is to say: for a given $\sigma$ a $b$ can be found to create dynamics which show the improvements. I report steady state $c^u_t/c^e_t$ because it does differ within this parameter space. I focus on the dynamics which lead to these observations, using impulse responses to illustrate and discuss the effects.

\(^{23}\) $C^u_t/C^e_t$ becomes dynamic under habit formation, as discussed in the model section, but I do not make any analysis of that in this report, and consumption does not itself appear in tables or diagrams in this paper.

\(^{24}\) The two cases are highlighted in Table 1 and Figures 1 and 2.
5.1 The Effect of Adding Habit Formation at Different Insurance Levels

Habit formation produces a 'hump-shaped' response to shocks. Since agents derive consumption utility relative to a reference level from the previous period, the response to shocks is delayed and smoothed. This effect of habit formation is known. In the absence of habit formation, the insurance effect appears as a change in the magnitude of fiscal-shock movements, by affecting the punishment associated with shirking, as described in Alexopoulos (2004).

Under technology shocks, lower insurance levels dampen and delay the initial positive wage effect by a shirking penalty effect as well. As Alexopoulos (2004) explains, $c_f^t$ increases more or less based on the relative magnitudes of investment increases and wage increases due to increased marginal product of labor.

Adding habit formation has the expected effect on responses to fiscal shocks, which can be seen in Figure 1. The response of employment to a technology shock is affected in a more surprising way, which is a consequence of the dynamics arising from imperfect effort monitoring. Adding habit formation in the presence of partial insurance has two effects which increase the initial wage response, and dampen the initial employment response, to the point of making it negative at high enough habit levels. First, the delayed investment response makes the shirking penalty effect on wages more pronounced earlier through its relative effect on initial $c_f^t$. Second, the shirking penalty effect itself is larger. The employer adjusts wages to keep effort constant, as before. Now, though, the constant $\chi$ includes previous consumption. $\chi_t = (c_t^f - b\bar{c}_{t-1}) / (c_t^a - b\bar{c}_{t-1})$, so with greater $b$ the initial wage increase is greater, to the point that the corresponding decrease in employment initially
overwhelms the increase due to the marginal product of labor.

The qualitative difference in the initial movement of $N$ under a technology shock drives the improvement (reduction) in $N-Y$ correlation at higher habit formation values. Illustrating this effect, Figure 3 and Table 2 depict the results under increasing habit formation. It is clear in the table that increasing habit formation reduces the $N-Y$ correlation. Viewing the figure makes it apparent that the source of this improvement is the initial drop in $N$. Figure 3 shows the transition to the negative initial response of $N$, and the trends in technology shock responses of $W$, $N$, and $Y$, with increasing habit formation. Table 2 corresponds to Figure 3 and should be viewed with it. The increasing negative $W-N$ correlation seems to arise from the same effect that decreases the $N-Y$ correlation and decreases $Y$ volatility. I do not show the fiscal shock responses since they follow the pattern already demonstrated in Figure 1.

5.2 Targeting Certain Improvements

Table 3 and Figure 4 should be viewed together. For different $b$, different values of $\sigma$ produce the $N-Y$ correlation and $Y$ volatility improvements. Increased volatilities of $N$ and $W$ come about in these ranges, as well as a large negative $W-N$ correlation, as can be predicted from viewing the impulse responses to the technology shock. The $Y$ volatility and the $N-Y$ correlation more closely match U.S. data than those the partial insurance model without habit formation produce. In Table 3 and the Figure 4 I have highlighted sets of values that bring us closer to the U.S. data in these respects. The dynamics of interest here seem to come from the technology shock effect on employment, so I do not show impulse responses
for fiscal shocks.

As has been mentioned, under increased habit formation a technology shock causes a sharper initial increase in wage. The wage increase corresponds to a lesser employment increase, eventually a decrease, under increasing habit formation values. For the range of values shown in Figure 4 and Table 3, an increased insurance level counteracts this effect. Thus, imperfect monitoring and habit formation interact to determine the initial response of wages and employment. Increasing insurance and increasing habit formation have opposing effects. Examination of Figure 4 makes this clear. There are multiple combinations of $\sigma$ and $b$ which yield second moments to match U.S. data, and they correspond to impulse responses that look very similar. These results appear at different $b$ levels given different $\sigma$ levels, and carry with them very similar volatility increases for $W$ and $N$, and high negative $W - N$ correlation. The best match to U.S. data is seen at parameter values highlighted in Table 3 and Figure 4, and the large negative $W - N$ correlation can easily be predicted from the impulse responses. However, the $c_t/c_t^*$ steady state does vary among parameterizations (see Table 3) so if we had some evidence for a certain consumption ratio then we would have some empirical support for a habit formation value.

6 Conclusion

By examining dynamic effects of habit formation and variable insurance in a shirking efficiency wage model, we can see effects of habit formation in a model with structural unemployment. Adding habit formation implies that the ratio of consumption of the unemployed to that of the employed is no longer constant. Examination of second moments, along with
impulse responses for technology and fiscal shocks, shows the interaction of habit formation and imperfect monitoring. Adding habit formation reduces output volatility and relaxes the high correlation of output and employment to a degree more consistent with observed data. However, a counterfactual negative correlation of wage and employment emerges, and the volatilities of both employment and wage increase, as compared to models such as the ones used by Alexopoulos (2004) and Givens (2008). Habit and insurance level pairs that cause the better match to U.S. data can imply different consumption ratios.

Perhaps one source of the inflated volatility of employment in my model comes from its failure to account for heterogeneity of workers’ income histories that would affect the IC constraints and create a wage dispersion effect, if employers have access to information about individual employment history. Although the model simulated in this paper ignores the possibility by assuming a single reference consumption level, we could expect a smoothing effect on employment variation, since lagged employment would then affect the IC constraints determining wages and employment. High (low) unemployment in the recent past drives down (up) optimum wages, and thus raises (lowers) present employment, dampening movements away from the steady-state level in either direction. A model allowing for this heterogeneity in wages\(^{25}\) may also yield a correlation of wage and employment closer to zero.

Empirical inquiries which might support the usefulness of this type of model would include formal testing of the fit of this model against other proposed sources of labor market friction. In addition, a study evaluating whether there is a wage premium associated with jobs that are less closely monitored for performance would lend some support to this type of model.

\(^{25}\)The model which allows differing IC constraints among workers, generating the wage dispersion, is work in progress.
Finally, although not directly related to the model presented in this paper, because I use external habit formation, any empirical demonstration of a wage differential resulting from differences in workers’ recent employment status could be taken to support a wage dispersion dynamic arising from heterogeneous work histories.
Appendix A  Deriving the incentive compatibility constraint

This function, contracted effort as a function of wage, constrains the contracted effort choice of the firm, appearing in section 2.2, Equation (16). Setting utility from working equal to utility expected from shirking, we arrive at a function for effort.

\[ U(c_t^e - b\tilde{c}_{t-1}, E_t) = dU(c_t^s - b\tilde{c}_{t-1}, 0) + (1 - d)U(c_t^e - b\tilde{c}_{t-1}, 0) \]

\[ \ln(c_t^e - b\tilde{c}_{t-1}) + \theta \ln(T - (hE_t + \zeta)) = \]

\[ d(\ln(c_t^s - b\tilde{c}_{t-1}) + \theta \ln(T)) + (1 - d)(\ln(c_t^e - b\tilde{c}_{t-1}) + \theta \ln(T)) \]

\[ \ln(c_t^e - b\tilde{c}_{t-1}) - (1 - d)(\ln(c_t^s - b\tilde{c}_{t-1}) - d(\ln(c_t^e - b\tilde{c}_{t-1}) = \theta \ln(T) - \theta \ln(T - (hE_t + \zeta)) \]

\[ \left( \frac{c_t^e - b\tilde{c}_{t-1}}{c_t^s - b\tilde{c}_{t-1}} \right)^d = \left( \frac{T}{T - (hE_t + \zeta)} \right)^\theta \]

\[ E_t = \Delta(w_t) = -\frac{T}{h} \left( \frac{c_t^e - b\tilde{c}_{t-1}}{c_t^s - b\tilde{c}_{t-1}} \right)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h} \]

I define \( \chi_t = (c_t^e - b\tilde{c}_{t-1})/(c_t^s - b\tilde{c}_{t-1}) \), equivalent to the ratio used by Alexopoulos (2004) only when habit formation is zero. \( c_t^e/c_t^s \) is not constant, but \( \chi_t \) is. So \( E_t = \Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h} \) represents effort as a function of the wage and exogenous constants. It can be used as a constraint on the firm’s problem.
Appendix B Deriving Equations (18) and (19)

An expression for wage is used in the demonstration that $\chi_t$ is constant. It comes directly from the consumption and insurance definitions. Section 2.2, Equation (18)

\[
c^*_t = c^f_t + sw_t h - F_t
\]
\[
c^r_t = c^f_t + w_t h - F_t
\]
\[
c^e_t - c^*_t = (1 - s)hw_t
\]
\[
hw_t = \frac{(c^e_t - b \tilde{c}_{t-1}) - (c^*_{t-1} - b \tilde{c}_{t-1})}{(1-s)}
\]
\[
(1 - s)^\frac{\chi_t}{\chi_{t-1}} hw_t = (c^e_t - b \tilde{c}_{t-1}).
\]

To derive the modified consumption ratio which is constant in a model with habit formation, we use consumption and insurance definitions exactly like those of Givens (2008), with the addition of habit formation to the definition of $\mu$. $\mu = 1/\chi$ and is a function of $\sigma$ and $b$. Section 2.2, Equation (19)

\[
(1 - s)^\frac{\chi_t}{\chi_{t-1}} hw_t = (c^e_t - b \tilde{c}_{t-1})
\]
\[
hw_t = \frac{\chi_{t-1}}{\chi_t} \frac{1}{(1-s)} (c^e_t - b \tilde{c}_{t-1})
\]
\[
f_t = \sigma(1 - N_t)hw_t
\]
\[
c^*_t = c^f_t + N_t\sigma hw_t
\]
\[
c^r_t = c^f_t + hw_t - f_t
\]
\[
c^e_t = c^f_t + shw_t - f_t
\]
\[
c^e_t = c^f_t + hw_t - \sigma(1 - N_t)hw_t
\]
\[
c^f_t = c^e_t - hw_t + \sigma(1 - N_t)hw_t
\]
\[
c^*_{t} = c^e_{t} - hw_{t} + \sigma(1 - N_{t})hw_{t} + N_{t}\sigma hw_{t}
\]
\[
c^*_t = c^e_t - \left(\frac{\chi_t}{\chi_{t-1}} \frac{1}{(1-s)} (c^e_t - b \tilde{c}_{t-1})\right) + \sigma(1-N_t)\left(\frac{\chi_{t-1}}{\chi_t} \frac{1}{(1-s)} (c^e_t - b \tilde{c}_{t-1})\right) + N_t\sigma \left(\frac{\chi_{t-1}}{\chi_t} \frac{1}{(1-s)} (c^e_t - b \tilde{c}_{t-1})\right)
\]
\[ c_t^u = c_t^e - \left( \frac{x_t^{-1}}{\chi_t} \left( \frac{1}{1-s} (c_t^e - b\bar{c}_{t-1}) \right) \right) + \sigma \left( \frac{x_t^{-1}}{\chi_t} \left( \frac{1}{1-s} (c_t^e - b\bar{c}_{t-1}) \right) \right) \]

\[ c_t^u = c_t^e - \frac{1-s}{1-s} \left( \frac{x_t^{-1}}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \]

\[ c_t^u - b\bar{c}_{t-1} = c_t^e - b\bar{c}_{t-1} - \frac{1-s}{1-s} \left( \frac{x_t^{-1}}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \]

\[ \frac{c_t^u - b\bar{c}_{t-1}}{c_t^e - b\bar{c}_{t-1}} = \mu(\sigma) = 1 - \frac{1-s}{1-s} \left( \frac{x_t^{-1}}{\chi_t} \right) \]

Notice this expression appears the same as that in Givens (2008), but in the case where \( b > 0 \), \( \chi_t \) is not the same. Thus, we have the expression relating the insurance level, the shirking detection probability, and the habit-modified consumption ratio \( \chi_t \), which is constant.
Appendix C  Deriving the Solow condition from first order conditions

Using the FOCs for $N$ and $W$, we can arrive at the Solow condition. This is the source of Equation (25), in section 2.2.

$\text{FOC } N_t$: $(1-\alpha)A_tK_t^\alpha(hE_tN_t)^{-\alpha}hE_t = w_th$

$\text{FOC } W_t$: $(1-\alpha)A_tK_t^\alpha(h\Delta(w_t)N_t)^{-\alpha}\Delta'(w_t) = 1$

\[
\Delta'(w_t) = \frac{1}{(1-\alpha)A_tK_t^\alpha(h\Delta(w_t)N_t)^{-\alpha}}
\]

\[
\frac{(w_t)\Delta'(w_t)}{\Delta(w_t)} = 1
\]

Appendix D  Demonstrating that $\chi_t$ is constant.

When $\chi_t$ is defined as $(c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$, it is constant. This finding is used in section 2.2. Under zero habit formation, this is exactly equivalent to the consumption ratio mentioned in Alexopoulos (2004) and calibrated to the value found by Gruber(1997). I recover steady-state $c_t^e/c_t^s$ and report it with the other results, after calibrating for $N = 0.941$.

We know $(1-s)\frac{\chi_t}{\chi_{t-1}}hw_t = (c_t^e - b\bar{c}_{t-1})$ and $\Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$. Using the Solow condition, $\frac{(w_t)\Delta'(w_t)}{\Delta(w_t)} = 1$, simply substitute, yielding

\[
(T - \zeta) \chi^{1+\frac{d}{\theta}} - T\chi (1-s) = T\frac{d}{\theta}(1-s\chi)(\chi - 1).
\]
References


Table 1. Simulated second moments and consumption ratios at different habit formation ($b$) and insurance ($\sigma$) values. Corresponds to Figures 1 and 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$c^u_t/c^e_t$</th>
<th>$\sigma_n$</th>
<th>$\sigma_y$</th>
<th>$\sigma_w$</th>
<th>$\rho(n,y)$</th>
<th>$\rho(n,w)$</th>
<th>$\rho(y,w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0.00$</td>
<td>$\sigma = 0.78$</td>
<td>0.7874</td>
<td>0.0168</td>
<td>0.0187</td>
<td>0.0063</td>
<td>0.9418</td>
<td>0.1318</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.89$</td>
<td>0.8930</td>
<td>0.0117</td>
<td>0.0159</td>
<td>0.0070</td>
<td>0.9147</td>
<td>0.4070</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1$</td>
<td>1.0000</td>
<td>0.0106</td>
<td>0.0152</td>
<td>0.0073</td>
<td>0.9028</td>
<td>0.4329</td>
</tr>
<tr>
<td>$b=0.20$</td>
<td>$\sigma = 0.78$</td>
<td>0.7744</td>
<td>0.0232</td>
<td>0.0218</td>
<td>0.0069</td>
<td>0.9547</td>
<td>-0.3439</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.89$</td>
<td>0.8231</td>
<td>0.0163</td>
<td>0.0180</td>
<td>0.0059</td>
<td>0.9461</td>
<td>0.1290</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1$</td>
<td>1.0000</td>
<td>0.0168</td>
<td>0.0186</td>
<td>0.0062</td>
<td>0.9437</td>
<td>0.1238</td>
</tr>
<tr>
<td>$b=0.40$</td>
<td>$\sigma = 0.78$</td>
<td>0.7912</td>
<td>0.0262</td>
<td>0.0179</td>
<td>0.0134</td>
<td>0.8795</td>
<td>-0.7731</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.89$</td>
<td>0.8231</td>
<td>0.0230</td>
<td>0.0193</td>
<td>0.0093</td>
<td>0.9179</td>
<td>-0.5622</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1$</td>
<td>1.0000</td>
<td>0.0233</td>
<td>0.0203</td>
<td>0.0089</td>
<td>0.9257</td>
<td>-0.5097</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>N/A</td>
<td>0.0087</td>
<td>0.0154</td>
<td>0.0111</td>
<td>0.8656</td>
<td>0.0739</td>
<td>0.2414</td>
</tr>
</tbody>
</table>

Notes: Simulated series are HP-filtered ($\lambda = 1600$). $c^u_t/c^e_t$, the ratio of unemployed to employed consumption, is recovered from the steady state if it cannot be calculated analytically ($\sigma \neq 1$). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1, 1964 through Q2, 2010, HP-filtered ($\lambda = 1600$).
Table 2. Simulated second moments and consumption ratios at different habit formation ($b$) values while holding insurance ($\sigma$) level constant. Corresponds to Figure 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$c_u^t/c_e^t$</th>
<th>$\sigma_n$</th>
<th>$\sigma_y$</th>
<th>$\sigma_w$</th>
<th>$\rho(n, y)$</th>
<th>$\rho(n, w)$</th>
<th>$\rho(y, w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0.26$</td>
<td>$\sigma = .78$</td>
<td>0.7784</td>
<td>0.0265</td>
<td>0.0227</td>
<td>0.0087</td>
<td>0.9482</td>
<td>-0.5702</td>
</tr>
<tr>
<td>$b=0.29$</td>
<td></td>
<td>0.7762</td>
<td>0.0286</td>
<td>0.0233</td>
<td>0.0101</td>
<td>0.9449</td>
<td>-0.6591</td>
</tr>
<tr>
<td>$b=0.32$</td>
<td></td>
<td>0.7743</td>
<td>0.0319</td>
<td>0.0238</td>
<td>0.0127</td>
<td>0.9375</td>
<td>-0.7571</td>
</tr>
<tr>
<td>$b=0.35$</td>
<td></td>
<td>0.7816</td>
<td>0.0288</td>
<td>0.0213</td>
<td>0.0126</td>
<td>0.9155</td>
<td>-0.7329</td>
</tr>
<tr>
<td>$b=0.38$</td>
<td></td>
<td>0.7795</td>
<td>0.0275</td>
<td>0.0181</td>
<td>0.0145</td>
<td>0.8791</td>
<td>-0.8045</td>
</tr>
<tr>
<td>$b=0.41$</td>
<td></td>
<td>0.7986</td>
<td>0.0254</td>
<td>0.0177</td>
<td>0.0128</td>
<td>0.8838</td>
<td>-0.7625</td>
</tr>
<tr>
<td>$b=0.44$</td>
<td></td>
<td>0.8013</td>
<td>0.0235</td>
<td>0.0153</td>
<td>0.0136</td>
<td>0.8350</td>
<td>-0.7861</td>
</tr>
<tr>
<td>$b=0.47$</td>
<td></td>
<td>0.8027</td>
<td>0.0218</td>
<td>0.0118</td>
<td>0.0148</td>
<td>0.7657</td>
<td>-0.8594</td>
</tr>
<tr>
<td>$b=0.50$</td>
<td></td>
<td>0.8046</td>
<td>0.0201</td>
<td>0.0091</td>
<td>0.0152</td>
<td>0.7034</td>
<td>-0.9057</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>N/A</td>
<td>0.0087</td>
<td>0.0154</td>
<td>0.0111</td>
<td>0.8656</td>
<td>0.0739</td>
<td>0.2414</td>
</tr>
</tbody>
</table>

Notes: Simulated series are HP-filtered ($\lambda = 1600$). $c_u^t/c_e^t$, the ratio of unemployed to employed consumption, is recovered from steady state if it cannot be calculated analytically ($\sigma \neq 1$). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ($\lambda = 1600$). At a $\sigma$ (insurance) value corresponding to Alexopoulos’s (2004) partial insurance model, trends associated with increasing $b$ (habit formation) values can be seen. Figure 3 shows the emergence of the initial $N$ response which creates the $N - Y$ correlation reduction seen here with increasing habit formation.
Table 3. Simulated second moments and consumption ratios at different habit formation ($b$) and insurance ($\sigma$) values. Corresponds to Figure 4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$c_t^u/c_t^e$</th>
<th>$\sigma_n$</th>
<th>$\sigma_y$</th>
<th>$\sigma_w$</th>
<th>$\rho(n,y)$</th>
<th>$\rho(n,w)$</th>
<th>$\rho(y,w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0.43$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma=.77$</td>
<td>0.7911</td>
<td>0.0237</td>
<td><strong>0.0142</strong></td>
<td>0.0146</td>
<td><strong>0.8156</strong></td>
<td>-0.8273</td>
<td>-0.3502</td>
</tr>
<tr>
<td>$\sigma=.86$</td>
<td>0.8117</td>
<td>0.0261</td>
<td>0.0191</td>
<td>0.0122</td>
<td>0.8995</td>
<td>-0.7297</td>
<td>-0.3582</td>
</tr>
<tr>
<td>$\sigma=.95$</td>
<td>0.8311</td>
<td>0.0242</td>
<td>0.0197</td>
<td>0.0103</td>
<td>0.9079</td>
<td>-0.6116</td>
<td>-0.2248</td>
</tr>
<tr>
<td>$b=0.48$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma=.77$</td>
<td>0.7992</td>
<td>0.0207</td>
<td>0.0100</td>
<td>0.0151</td>
<td>0.7279</td>
<td>-0.8906</td>
<td>-0.3366</td>
</tr>
<tr>
<td>$\sigma=.86$</td>
<td>0.8161</td>
<td>0.0227</td>
<td><strong>0.0142</strong></td>
<td>0.0138</td>
<td><strong>0.8168</strong></td>
<td>-0.8036</td>
<td>-0.3131</td>
</tr>
<tr>
<td>$\sigma=.95$</td>
<td>0.8403</td>
<td>0.0224</td>
<td>0.0168</td>
<td>0.0113</td>
<td>0.8715</td>
<td>-0.6846</td>
<td>-0.2397</td>
</tr>
<tr>
<td>$b=0.53$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma=.77$</td>
<td>0.8107</td>
<td>0.0190</td>
<td>0.0075</td>
<td>0.0151</td>
<td>0.6590</td>
<td>-0.9273</td>
<td>-0.3298</td>
</tr>
<tr>
<td>$\sigma=.86$</td>
<td>0.8095</td>
<td>0.0200</td>
<td>0.0080</td>
<td>0.0157</td>
<td>0.6755</td>
<td>-0.9283</td>
<td>-0.3537</td>
</tr>
<tr>
<td>$\sigma=.95$</td>
<td>0.8412</td>
<td>0.0236</td>
<td><strong>0.0148</strong></td>
<td>0.0140</td>
<td><strong>0.8270</strong></td>
<td>-0.8061</td>
<td>-0.3341</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>N/A</td>
<td>0.0087</td>
<td><strong>0.0154</strong></td>
<td>0.0111</td>
<td><strong>0.8656</strong></td>
<td>0.0739</td>
<td>0.2414</td>
</tr>
</tbody>
</table>

Notes: Bold numbers indicate $b$ (habit formation) and $\sigma$ (insurance) values that correspond to the bold-bordered diagrams in Figure 4. Simulated series are HP-filtered ($\lambda = 1600$). $c_t^u/c_t^e$, the ratio of unemployed to employed consumption, is recovered from the steady state if it cannot be calculated analytically ($\sigma \neq 1$). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ($\lambda = 1600$). The opposed effects of the habit and insurance levels are apparent here and in Figure 4. Note how $c_t^u/c_t^e$ varies even where the simulation is very similar with respect to deviations and correlations.
Notes: Varying $b$ (habit formation) and $\sigma$ (insurance) values yield qualitatively different responses. These diagrams correspond to Table 1. Compare the top, first and third diagrams to Alexopoulos’s (2004) partial and full insurance setups. Fiscal shocks are not pictured in figures after this one since the interesting dynamics arise from the technology shocks.
Figure 2: Impulse response from technology shock

Notes: Varying $b$ (habit formation) and $\sigma$ (insurance) values yield qualitatively different responses. These diagrams correspond to Table 1. Compare the top, first and third diagrams to Alexopoulos’s (2004) partial and full insurance setups.
Figure 3: Impulse response from a technology shock at increasing habit formation ($b$) levels, at an insurance ($\sigma$) level corresponding to Alexopoulos’s (2000) partial insurance setup.

Notes: A progressively deeper initial drop in $N$ arises with increased habit formation. Also visible is the lengthening period until the peak response. These effects contribute to the pattern of reduced $\rho(n, y)$ with increasing habit formation seen in Table 2. A pattern of increasing (counterfactual) negative $\rho(n, w)$ arises from the wage and employment responses.
Figure 4: Impulse response from a technology shock. Similar dynamics appear at different habit formation ($b$) values for different insurance ($\sigma$) values.

Notes: Bold boxes correspond with bold numbers in Table 3. While many dynamics are nearly identical, the implied consumption ratio, unemployed to employed, $c_t^u/c_t^e$, differs.