PHYS 3160 HOMEWORK ASSIGNMENT 01

DUE DATE JANUARY 03, 2020

Instructor: Dr. Daniel Erenso Name: ______________________________

Mandatory problems: 1 & 2

Student signature:____

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1. Find and solve the Euler-Lagrange equation that make the following integrals stationary.

(a)

$$I = \int_{t_1}^{t_2} \sqrt{t}\sqrt{1+\dot{q}^2} dt, \tag{1}$$

where t is the independent variable and $\dot{q} = \frac{dq}{dt}$.

(b)

$$I = \int_{y_1}^{y_2} \left(x^2 + x'^2\right) dy \tag{2}$$

here y is the independent variable and $x' = \frac{dx}{dy}$.

(b)

$$I = \int_{\theta_1}^{\theta_2} \theta \left(\sqrt{1-r'^2}\right) d\theta, \tag{3}$$

θ is the independent variable and $r' = \frac{dr}{d\theta}$.

2. From introductory physics you know that light travels with a speed, $c = 3 \times 10^8 m/s$, in vacuum. When it travels in a medium with a refractive index, $n > 1$ the light slows down and the speed, v is given by

$$v = \frac{c}{n}. \tag{4}$$

You also know that when light travels from one medium with refractive index $n_1$to another with refractive index, n_2, the light could bend towards or away from the normal depending on which refractive index is greater or less (Fig. 1). Using Calculus of variation show that,

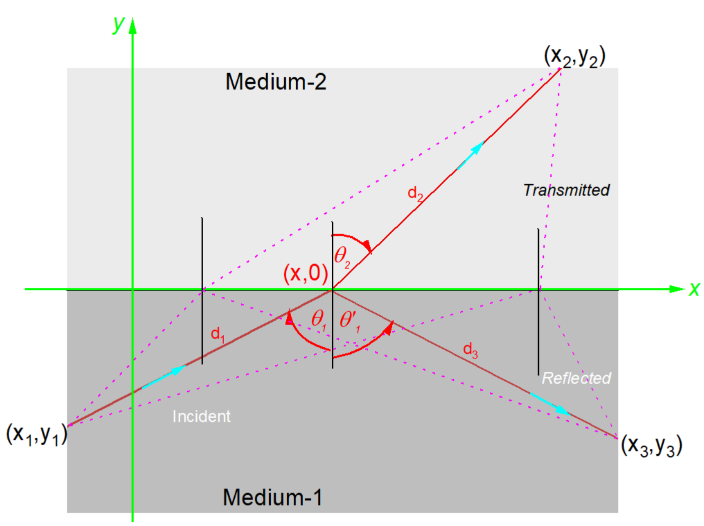

Figure 1: Different straight-line paths for the light traveling from point1 to point 2. The incident light partly gets reflected and partly gets refracted. The shortest paths are defined by the law of reflection and law of refraction.

(a) The angle of incidence, θ_1, and angle of transmission, θ_2, are related by the law of refraction (Snell's law)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \tag{5}$$

Hint:Minimize the time taken by the light

$$I = \int_1^2 dt, \tag{6}$$

Note that dt is an infinitessimal time for the light to travel an infinitessimal distance ds along the path of the light.

(b) The angle of incidence, θ_1, and the angle of reflection, θ_1',are related by the low of reflection

$$\theta_1 = \theta_1'. \tag{7}$$

Hint:Minimize the time taken by the light

$$I = \int_1^3 dt. \tag{8}$$

N.B. In either case the light travels in a straight line.

3. *This is a sort of reading assignment in my note or in the text.* Starting from

$$I(\epsilon) = \int_{x_1}^{x_2} F(x, Y, Y')\, dx,$$

where

$$Y(x) = y(x) + \epsilon\eta(x),$$

derive the *Euler-Lagrange Equation.*

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0,$$

by applying calculus of variation.

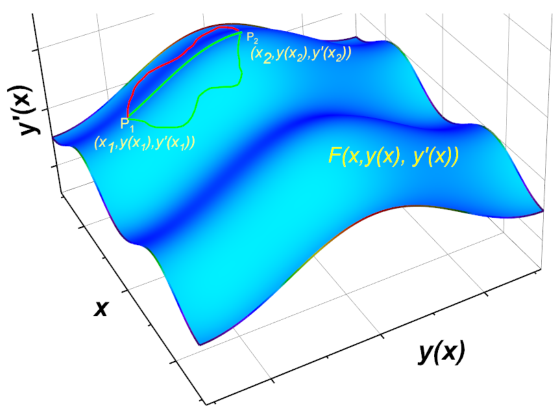

Figure 2: A surface defined by the function $F(x, y(x), y'(x) = \frac{dy}{dx})$

4. Re-do Example 9.2 (in my note) applying Euler-Lagrange Equation,

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0,$$

or

$$\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x'}\right) - \frac{\partial F}{\partial x} = 0,$$

whichever is easier.

5. Consider a surface of revolution generated by revolving a curve $y(x)$ about the x-axis. The curve is required to pass through fixed end points (x_1, y_1) and (x_2, y_2) as shown in Fig. 3. Find the curve $y(x)$ that gives the minimum area for the resulting surface using the *Euler-Lagrange Equation.*

6. The speed of an electromagnetic wave in an atmosphere increases in proportion to the height, $v(y) = y/b$, where $b > 0$ is some parameter describing the speed of the electomagnetic wave. This shows that, for example, the speed becomes zero when the height is zero (i.e. $y = 0 \Rightarrow v = 0$) which simulates the condition at the surface of a black hole, called its *even horizon*, where the gravitational force is so strong that the velocity of light goes to zero, thus even trapping the light. Find the optical path in an atmosphere. *Hint: the optical path must be a shortest path and do not expect a straight line.*

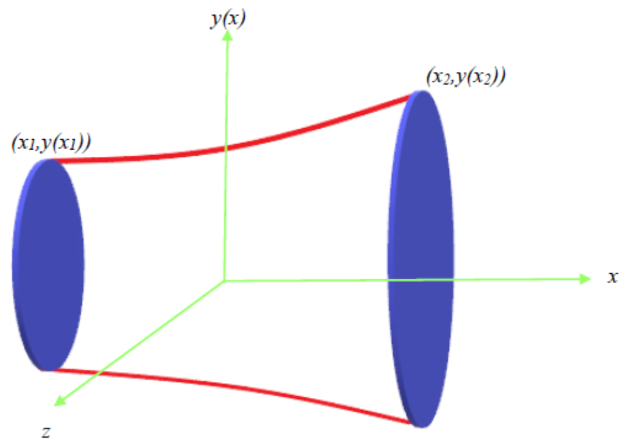

Figure 3: A curve rotated about the x-axis.