PHYS 3160 HOMEWORK ASSIGNMENT 02

DUE DATE February 10, 2020

Instructor: Dr. Daniel Erenso Name: ____________________

Mandatory problems: 4, 6 (b) & (d)

Student signature:____

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1. In Example 9.4 using *the Euler-Lagrange equation we had solved the brachystochrone problem, assuming the "material point" starts from rest, . We did show that in fact the shortest time if defined by an equation of an inverted cycloid*

$$x = \frac{1}{2c}(\theta - \sin(\theta)), y = -\frac{1}{2c}(1 - \cos(\theta)). \tag{1}$$

Show that when the material point starts with some initial velocity, v_0, still the shortest time path is defined by an inverted cycloid.

2. Consider the motion of a mass m moving under the influence of a central force (that is, a force acting only along the radial direction) given by

$$\vec{F} = -f(r)\hat{r} \tag{2}$$

for some function $f(r)$. Assume that the motion is confined to a plane and the position of the mass can be described using polar coordinates (r, φ)

$$\vec{r} = r\cos(\varphi)\hat{x} + r\sin(\varphi)\hat{y} \tag{3}$$

where $\varphi = 0$,as shown in Fig.1

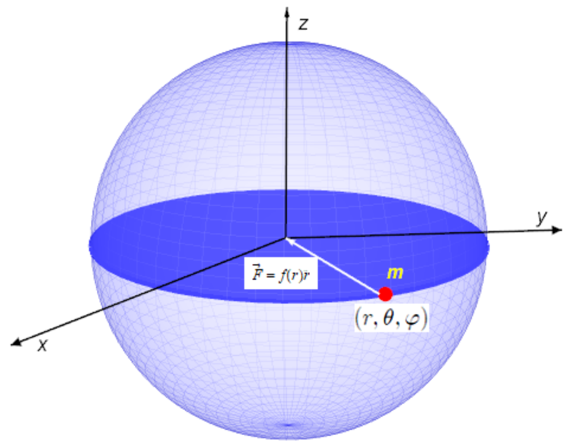

Figure 1: A mass m under a central force motion confined to the x-y plane.

(a) Find the Lagrangian

(b) Using Euler-Lagrange equation find the equation of motion for the mass for the radial and angular coordinates (i.e. r and φ). Show that one of these equations gives you the law of conservation of angular momentum, $\vec{L}$,

$$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = 0 \Rightarrow \vec{L} = I\vec{\omega} = \text{Constant}, \tag{4}$$

where $I = mr^2$ is the moment of inertia.

(c) For the case in which the radial distance is a constant

$$\dot{r} = \frac{dr}{dt} = 0 \tag{5}$$

you will find the equation of motion for a mass, m, moving in circle

$$mr\dot{\theta}^2 = -f(r) \Rightarrow \frac{m\left(r\dot{\theta}\right)^2}{r} = -f(r) \Rightarrow \frac{mv^2}{r} = -f(r), \tag{6}$$

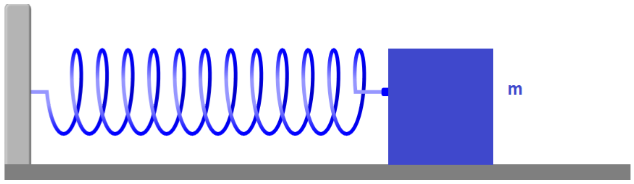

Figure 2: A harmonic oscillator in a horizontal plane.

3. *A one-dimensional harmonic oscillator*: Consider a mass, m, attached to one end of a spring with spring constant, k, . Find the Lagrangian and the equation of motion for the mass using the Euler-Lagrange equation for the following two cases.

(a) The other end of the spring is attached to a wall as shown in Fig. and the mass is oscillating on a frictionless table.

(b) The other end of the spring is attached to a ceiling as shown in Fig.3 and the mass is oscillating in a vertical plane.

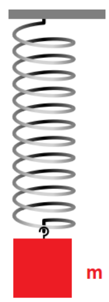

Figure 3: A harmonic oscillator in a vertical plane.

4. Consider two masses, m_1 mass m_2, connected by a string of length, l, with negligible mass. The string passes through a hole at the center of a table . The mass m_1 is on the table and it can move on the table. The surface of the table is frictionless. The second mass, m_2, hanging from the other end of the string can move up or down on a vertical plane. (see Fig. 4.

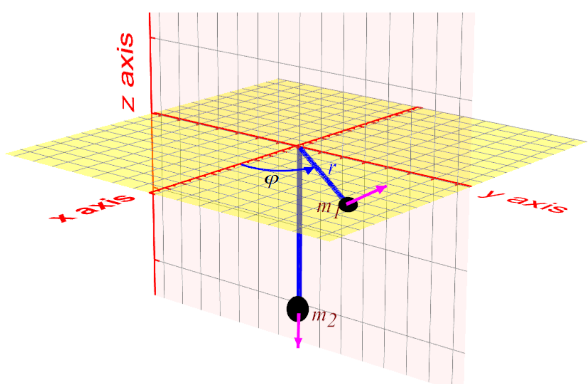

Figure 4: Two masses connected by a string of length l. In cylindrical coordinates the position for m_1 is $(r, \varphi, 0)$ and for m_2 is $(0, 0, -z_2)$. Note that $z_2 + r = l$.

(a) Using cylindrical coordinates (r, φ, z), find the Lagrangian

(b) Find the equation of motion for the two masses using Euler-Lagrange equation.

5. Find the magnitude of the vector pointing from point P to point Q, when these points are

(a) $P = (4, -1, 2, 7)$ and $Q = (2, 3, 1, 9)$.

(b) $P = (-1, 5, -3, 2, 4)$ and $Q = (2, 6, 2, 7, 6)$.

(c) Points described by the Minkowski space-time coordinates, $P = (x_1, y_1, z_1, ct_1)$ and $P = (x_2, y_2, z_2, ct_2)$ where c is the speed of light in free space. *You will see this in General relativity.*

6. For the matrices listed (a)-(d)

i. Find the eigenvalues and eigen vectors

ii. Construct the matrix C that diagonalizes each these matrices and determine its inverse matrix, C^{-1}

iii. Compute $C^{-1}MC$ for each matrices.(*this part may be done using Mathematica, in which case appropriate output must be provided*)

iv. Show that the matrices in (b) and (d) are Hermitian.

(a)

$$M = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}, \tag{7}$$

(b)

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, M = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{8}$$

These matrices are usually represented as σ_x, σ_y, and σ_z and are known as the Pauli Spin-1/2 matrices that you will see in quantum mechanics.

(c)

$$M = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

(d)

$$M = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{10}$$

This matrix is also related to spin-matrices (but for spin-1 particles) and you will also see it in quantum mechanics.