PHYS 3160 HOMEWORK ASSIGNMENT 03

DUE DATE FEBRUARY 17, 2020

Instructor: Dr. Daniel Erenso Name: ________________________

Mandatory problems: 1 & 3

Student signature:____

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P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

1. Consider a system consisting of two masses m_1 and m_2 connected by three three springs with spring constant k_1, k_2, and k_2 as shown in Fig. 1.

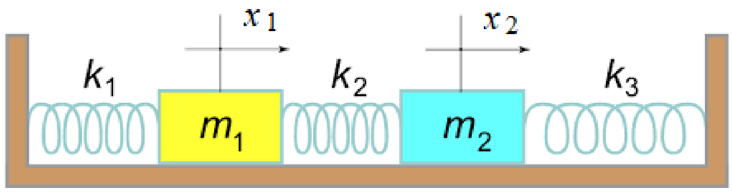

Figure 1: Two masses and three different springs.

The masses can slide on a horizontal, frictionless surface. The springs are at their unstretched/uncompressed lengths when the masses are at its equilibrium positions. At $t = 0$, the masses are displaced from its equilibrium positions by the amounts x_{10} and x_{20} and released from rest.

(a) Find the kinetic energy, the potential energy, and the Lagrangian. **Using the Euler-Lagrange equation** derive the equations of motion for each masses and express the equations using matrices

$$\left[\begin{array}{c} \ddot{x}_1 \\ \ddot{x}_2 \end{array}\right] = M \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right]. \tag{1}$$

(b) Let's assume that two atoms have nearly the same mass (i.e. $m_1 \simeq m_2 = m$) and ,

$$k_1 = 5k, k_2 = 2k, k_3 = 2k. \tag{2}$$

Using Similarity Transformation find the Eigenvalues and Eigenvectors for the matrix M.

(c) For the two masses find the displacements ($x_1(t)$ and $x_2(t)$) and speeds ($\dot{x}_1(t)$ and $\dot{x}_2(t)$)

(e) Find the propagator matrix.

(f) Describe the Normal Modes of Vibration of the atoms.

2.

(a) Prove that

$$B(q, p) = B(p, q) \tag{3}$$

(b) Express the integrals

$$I_1 = \int_0^1 \frac{x^4}{\sqrt{1 - x^2}} dx, \quad I_2 = \int_0^\pi \sin^3(\theta) \cos(\theta) d\theta \tag{4}$$

as beta functions and then write each beta functions in terms of the Gamma functions using the relation we derived in *Example 6.2*,

$$B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p + q)}. \tag{5}$$

When possible use the Gamma function formulas such as

$$\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx, \quad \Gamma(p + 1) = p\Gamma(p), \; \Gamma(1/2) = \sqrt{\pi} \tag{6}$$

to write an exact answer in terms of π, $\sqrt{2}$, etc.

(c) Applying the result in *Example 11.1* show that the integral

$$\int_{-\infty}^{\infty} e^{-x^2/a} dx = \sqrt{a\pi}, \tag{7}$$

for $a > 0$.

3. Using Stirling's formula evaluate

(a)

$$\lim_{n\to\infty}\left[\frac{\Gamma\left(n+\frac{3}{2}\right)}{\sqrt{n}\Gamma\left(n+1\right)}\right] \tag{8}$$

(b)

$$\lim_{n\to\infty}\left[\frac{(2n)!\sqrt{n}}{2^{2n}(n!)^2}\right] \tag{9}$$

4. The integral

$$\int_x^\infty u^{p-1}e^{-u}du = \Gamma(p,x) \tag{10}$$

is called an *incomplete Gamma function.* Note that for $x = 0$, you find the Gamma function

$$\int_0^\infty u^{p-1}e^{-u}du = \Gamma(p). \tag{11}$$

By repeated integration find several terms of the asymptotic series for $\Gamma(p, x)$.

NB: I found

$$\begin{aligned}\Gamma(p,x) &= \int_x^\infty u^{p-1}e^{-u}du \\ &= x^{p-1}e^{-x}\left[1+(p-1)\,x^{-1}+(p-1)\,(p-2)\,x^{-2}+(p-1)\,(p-2)\,(p-3)\,x^{-3}...\right]\end{aligned} \tag{12}$$

5. Using the Gamma and Beta function formulas show that

$$\int_0^\infty \frac{dy}{(1+y)\sqrt{y}} = \pi \tag{13}$$

6.

(a) Prove that the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{14}$$

is an odd function.

(b) Show that

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x e^{-t^2/2}dt = \frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(x/\sqrt{2}\right), \tag{15}$$

where

$$\operatorname{erf}\left(x/\sqrt{2}\right) = \frac{2}{\sqrt{\pi}}\int_0^{x/\sqrt{2}} e^{-t^2}dt \tag{16}$$

is the error function.