PHYS 3160 HOMEWORK ASSIGNMENT 04

DUE DATE FEBRUARY 24, 2020

Instructor: Dr. Daniel Erenso Name: ________________________

Mandatory problems: 2 & 5

Student signature:____

Comment:____
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P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

1. For the same pendulum in Example 11.5

(a) Use the Euler-Lagrange equation to find the equation of motion for the mass, m.

(b) The resulting equation is a none-linear differential equation. Show that this equation for small amplitude of oscillation gives a homogenous linear second order differential equation. By solving this equation show that the period of oscillation is given by same expression.

$$T = 2\pi\sqrt{\frac{l}{g}}, \tag{1}$$

2. Prove the relations

$$\delta(x) = \delta(-x), \delta(ax) = \frac{1}{a}\delta(x),$$

3. Show that

$$\int_{-\infty}^{\infty} \frac{d\delta(x)}{dx} f(x)\, dx = -\left.\frac{df(x)}{dx}\right|_{x=0} \tag{2}$$

4. Form introductory physics, the electric potential, $V(\vec{r})$, due to a point charge located at the origin $(0,0,0)$ (i.e. $r=0$) at a point in space described by the position vector, $\vec{r}$, is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}. \tag{3}$$

Show that the volume charge density, $\rho(\vec{r})$, for this point charge can be expressed in terms of the Dirac delta function

$$\rho(\vec{r}) = \frac{dq}{d\tau} = q\sigma(\vec{r}) = q\sigma(x)\,\sigma(y)\,\sigma(z), \tag{4}$$

where dq an infinitessimal charge in an infinitesimal volume $d\tau$.

5. The volume charge density, $\rho(\vec{r})$, of a point charge, q, placed at a point on the x-axis, $\vec{r}_0 = a\hat{x}$, can be expressed as

$$\rho(\vec{r}) = q\sigma(\vec{r} - \vec{r}_0), \tag{5}$$

where $\sigma(\vec{r})$ is the Dirac Delta function. Show that the electric potential, $V(\vec{r})$, due to this point charge is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r} - \vec{r}_0|} = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(x-a)^2 + y^2 + z^2}}. \tag{6}$$

The electric potential for a volume charge distribution is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|}, \tag{7}$$

where $\vec{r}'$ is the position of the infinitesimal charge $dq' = \rho(\vec{r}')\, d\tau'$, in an infinitesimal volume $d\tau'$, and $\rho(\vec{r}')$ is the charge density in the volume V.

6. Show that

$$\delta[(x - x_1)(x - x_2)] = \frac{\delta(x - x_1) + \delta(x - x_2)}{x_1 - x_2} \tag{8}$$