PHYS 3160 HOMEWORK ASSIGNMENT 05
DUE DATE MARCH 02, 2020

Instructor: Dr. Daniel Erenso Name: ______________________________

Mandatory problems: 1 & 5

Student signature:____

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P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

1. For the following differential equations find solutions using (a) series substitution method and (b) elementary method (any of the methods you were introduced in Theoretical Physics I: Part I.

(a)

$$\frac{dx(t)}{dt} - 3t^2 x(t) = 0 \tag{1}$$

(b)

$$\frac{d^2x(t)}{dt^2} - 2\frac{dx(t)}{dt} + x(t) = 0 \tag{2}$$

2. Show that the set of functions defined by

$$f_n(t) = \exp\left(\frac{int}{l}\right) \tag{3}$$

for $n = 0, \pm1, \pm2, \pm3...$ form an orthogonal set of functions for all $t \;\varepsilon\; (-l, l)$. If we want to make these set of functions an orthonormal set of functions, what must be the normalization factor.

3. We have shown that the solution to the Legendre differential equation

$$\left(1 - x^2\right) y'' - 2xy' + l(l+1)y = 0, \tag{4}$$

is given by

$$\begin{aligned} y_e(x) = a_0 \bigg\{ 1 &- \frac{l(l+1)}{2!}x^2 + \frac{l(l+1)(l-2)(l+3)}{4!}x^4 \\ &- \frac{l(l+1)(l-2)(l+3)(l-4)(l+5)}{6!}x^6 + ... \bigg\} \end{aligned} \tag{5}$$

for even l and by

$$\begin{aligned} y_o(x) = a_1 \bigg\{ x &- \frac{(l-1)(l+2)}{3!}x^3 + \frac{(l-1)(l+2)(l-3)(l+4)}{5!}x^5 \\ &- \frac{(l-1)(l+2)(l-3)(l+4)(l-5)(l+6)}{7!}x^7 + ... \bigg\} \end{aligned} \tag{6}$$

for odd l. Using these equations find the solutions for $l = 4$ and 5. By imposing the condition that $y(x)$ should be one at $x = 1$, determine the constants a_0 and a_1 in each case and write the expressions for *the Legendre polynomials*, $P_4(x)$ and $P_5(x)$. Compare the results with the results in the note or with results you determined using computer.

4.

(a) Using Leibniz' rule

$$\frac{d^N}{dt^N}(uv) = \sum_{n=0}^{N} \begin{pmatrix} N \\ n \end{pmatrix} \left(\frac{d^n u}{dt^n}\right)\left(\frac{d^{N-n} v}{dt^{N-n}}\right) \tag{7}$$

find the derivative

$$f(t) = \frac{d^6}{dt^6}\left(t^2 \sin(t)\right). \tag{8}$$

(b) Use the Rodrigues' formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2 - 1\right)^n \tag{9}$$

to determine $P_n(x)$ for $n = 0, 1, 2, 3,$ and 4. Check your result with what I have in the Note.

5.

(a) The Legendre Polynomials can be generated using

$$P_l(x) = \frac{1}{l!} \frac{d^l}{dh^l} \phi(x, h) \bigg|_{h=0}, \tag{10}$$

for the function

$$\phi(x, h) = \left(1 - 2xh + h^2\right)^{-1/2}. \tag{11}$$

Show this is true for $l = 3$ and $l = 4$.

(b) Use the recursion relation

$$lP_l(x) = (2l - 1)\, xP_{l-1}(x) - (l - 1)\, P_{l-2}(x)$$

and the values of $P_0(x)$ and $P_1(x)$ to find $P_l(x)$ for $l = 2, 3, 4, 5,$ and 6.

(c) You often see Legendre polynomials in Electricity and magnetism. Such as in the description of charge distribution on a aspherical surface. For example, a surface charge density could be expressed as

$$\sigma = \frac{q}{4\pi R^2} \left[\cos(\theta) - \cos^3(\theta)\right], \tag{12}$$

where q is the total charge on the surface and R is the radius of the sphere. Suppose we replace, $x = \cos(\theta)$, the charge density can be expressed as

$$\sigma = \frac{q}{4\pi R^2} \left(x - x^3\right). \tag{13}$$

Express this charge density using Legendre polynomials.

6.

(a) Re-drive the recursion relation for the Legendre polynomials that is derived in the note

(b) Use *Leibniz' Rule* to find

$$\frac{d^8}{dx^8} \left(x^2 e^{2x}\right). \tag{14}$$

Note: Both (a) and (b) have been worked out in the lecture note. Do not directly copy. Try to understand how it is done and do it on your own.