PHYS 3160 HOMEWORK ASSIGNMENT 06

DUE DATE MARCH 18, 2020

Instructor: Dr. Daniel Erenso Name: ______________________

Mandatory problems: 3, 5,6, &7

Student signature:_______________________________________

Comment:____
____
____

P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

1. Show that
$$\int_{-1}^{1} P_l(x)\,dx = 0 \text{ for } l > 0 \tag{1}$$

2. For the following functions
$$f(x) = \begin{cases} \cos(nx), & \text{for } x\ \varepsilon\ (-\infty,\infty), \\ e^{-x^2/2}, & \text{for } x\ \varepsilon\ (-\infty,\infty), \\ P_2(x) = \frac{1}{2}\left(3x^2-1\right), & \text{for } x\ \varepsilon\ (-1,1), \end{cases} \tag{2}$$

(a) For the given intervals evaluate the integral
$$I = \int f^*(x)\,f(x)\,dx. \tag{3}$$

(b) State the normalized functions for each.

3. For the function
$$f(x) = \begin{cases} 0, & -1 < x < 0, \\ x, & 0 < x < 1, \end{cases} \tag{4}$$

(a) find the first four none zero terms in the Legendre series,

(b) using computer (Mathematica) find these terms,

(c) plot the graph Legendre series as function of x and by considering different number of terms and show that as the number of terms increases the function becomes closer and closer to the exact function plot.

4. Using the transformation of variable defined by
$$x = \cos\theta, \tag{5}$$
show that the differential equation
$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dy(\theta)}{d\theta}\right) + \left(l(l+1) - \frac{m^2}{\sin^2\theta}\right)y(\theta) = 0. \tag{6}$$
can be written as
$$\left(1-x^2\right)\frac{d^2y(x)}{dx^2} - 2x\frac{dy(x)}{dx} + \left(l(l+1) - \frac{m^2}{1-x^2}\right)y(x) = 0 \tag{7}$$

5. Using the differential equation
$$\left(1-x^2\right)\frac{d^2P_l^m(x)}{dx^2} - 2x\frac{dP_l^m(x)}{dx} + \left[l(l+1) - \frac{m^2}{1-x^2}\right]P_l^m(x) = 0. \tag{8}$$
and integration by parts, show that
$$\int_{-1}^{1} P_l^m(x)\,P_n^m(x)\,dx = 0, \text{ for } l \neq n. \tag{9}$$

6. Show that the length of the vector, $\vec{d} = \vec{r} - \vec{r'}$, (see Fig. 1) between the two points described by the vectors, $\vec{r}$ and $\vec{r'}$, is given by
$$|\vec{r} - \vec{r'}| = r^2 + r'^2 - 2rr'\cos(\gamma), \tag{10}$$
where
$$\cos(\gamma) = \cos(\theta)\cos\left(\theta'\right) + \sin(\theta)\sin\left(\theta'\right)\cos\left(\varphi - \varphi'\right). \tag{11}$$

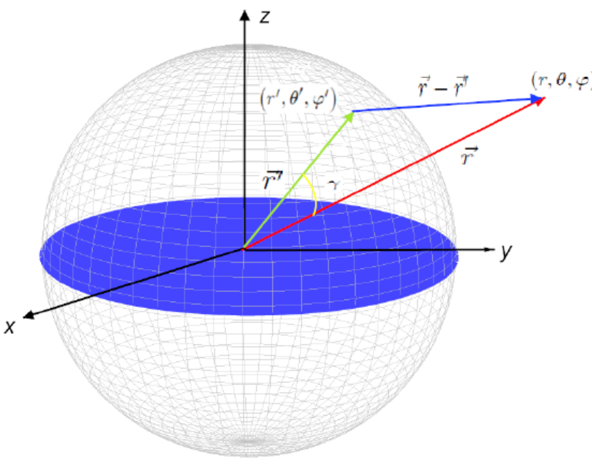

Figure 1: Two point described by two vectors, $\vec{r}$ and $\vec{r}'$. In spherical coordinates the two points have coordinates (r, θ, φ) and (r', θ', φ') ,respectively. The angle between these two vectors is γ.

7. A spherical shell of radius R has a constant surface-charge density, σ. The sphere is centered at the origin of coordinates. The point P is a distance, $r = d < R$, from the center of the sphere. Find an expression for the electrostatic potential at the point P, due to the charged sphere.Note that

$$\sigma = \frac{dq'}{da} = \text{Constant},$$

where da is an infinitessimal area on the surface of the sphere which we can express in spherical coordinates as

$$da = R^2 \sin(\theta)\, d\theta d\varphi,$$

R is the radius of the sphere.

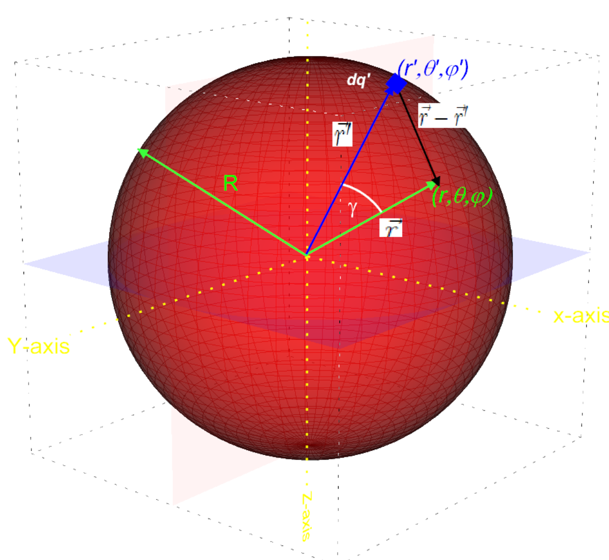

Figure 2: A spherical shell carrying a constant surface charge density, σ.

8. The spherical harmonics form a complete set and therefore any function $f(\theta, \varphi)$ defined for $0 < \theta < \pi$ and

$0 < \varphi < 2\pi$, can be expressed as "spherical harmonics series"

$$f(\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{-l}^{l} A_{lm} Y_{lm}(\theta,\varphi), \tag{12}$$

where

$$A_{lm} = \int_0^{\pi} \int_0^{2\pi} Y_{lm}^{*}(\theta,\varphi) f(\theta,\varphi) \sin(\theta) \, d\theta d\varphi. \tag{13}$$

Applying this relation show that the function

$$f(\theta,\varphi) = P_l(\cos(\gamma)) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\theta,\varphi) Y_{lm}^{*}(\theta',\varphi'), \tag{14}$$

where

$$\cos(\gamma) = \cos(\theta)\cos(\theta') + \sin(\theta)\sin(\theta')\cos(\varphi - \varphi'). \tag{15}$$