PHYS 3160 Midterm Exam

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Instructions and important notes

- The problems listed are review problems for the in-class part of the exam.
- Some of the problems require a specific method to solve. You must use the specific methods stated in the problem.
- Please pay attention to italicized or bold phrases.
- To receive full credit, your work must be clear and complete.
- Begin the solution of each problem on a new page. Do not use the back pages!
- The solutions to each problems must be presented in order.
- You must box in the final result to each part of the problems when it is appropriate.
- You must attach the cover page of this exam on top of the pages of your properly ordered solutions to Part II.

YOU HAVE ONE HOUR AND TWENTY FIVE MINUTES TO COMPLETE PART I

Problem	1	2	3	Total
Score	/15	/15	/20	/100

Part I: In-class test review problems

1. *The Atwood's Machine: A string of length, l, passes over a frictionless pulley connecting two masses, m_1 and m_2.*

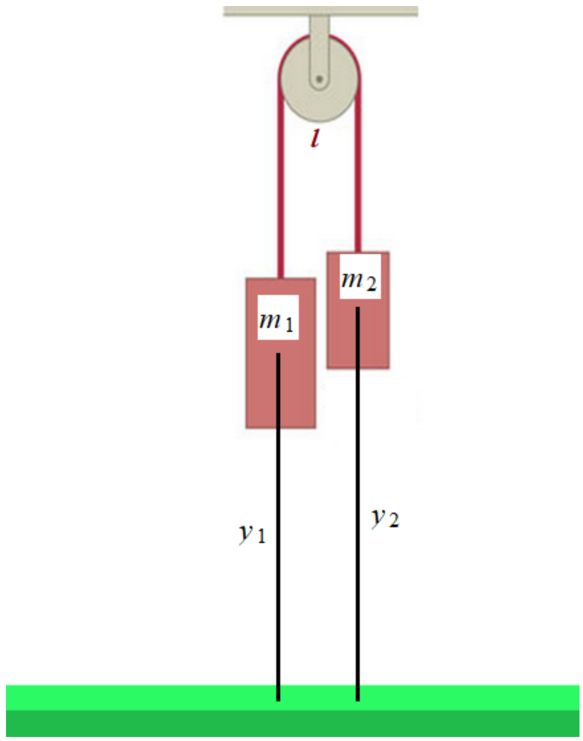

Figure 1: *Atwood's Machine.*

(a) Find the Lagrangian.

(b) Use Euler-Lagrange equation to find the acceleration for the two masses.

2. *Consider a simple pendulum with a mass m suspended from the end of a light rigid rod of length l. We pull the pendulum to the side by an angle, α, and release it from rest. Show that the period of the pendulum, T, is given by*

$$T = 4\sqrt{\frac{l}{g}}K\left(m\right)$$

where

$$K\left(m\right) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - m\sin^2\varphi}}, \tag{1}$$

is the complete elliptic integral of the first kind *Assume that $\theta = 0$ and $d\theta/dt > 0$ at $t = 0$, where θ is the angle of the pendulum from the vertical. Find the series expansion for the period and show that the period for a small amplitudes of motion is given by*

$$T \simeq 2\pi\sqrt{\frac{l}{g}}, \tag{2}$$

3. Consider a surface of revolution generated by revolving a curve $y\left(x\right)$ about the x-axis. The curve is required to pass through fixed end points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$. Find the curve $y\left(x\right)$ that gives the minimum area for the resulting surface using the *Euler-Lagrange Equation.*

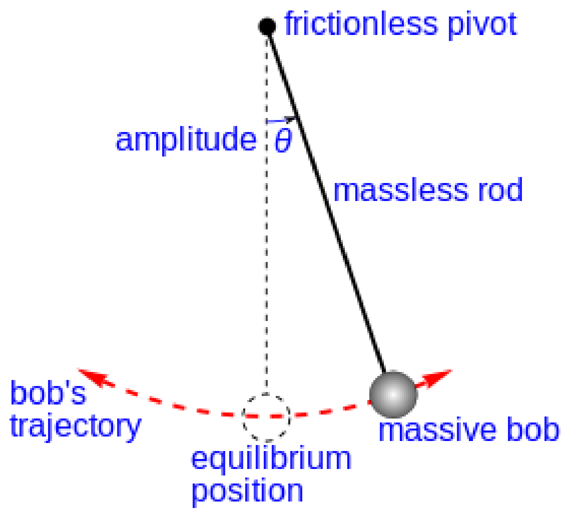

Figure 2: A simple pendulum. At the initial time, $t = 0$, the mass m was displaced by an angle, α, from the vertical.

4. Consider a mass, m, moving under the influence of a central force (that is, a force acting only along the radial direction) given by

$$\vec{F} = f(r)\hat{r}$$

for some function $f(r)$. Assume that the motion is confined to the x-y plane.

(a) Find the potential energy using polar coordinates (r, θ)

(b) Determine the Lagrangian.

(c) Use Euler-Lagrange equation to find the equation of motion in polar coordinates. (You are not expected to solve these equations)

(d) Make a physical interpretation of these equations.

(e) For

$$\dot{r} = \frac{dr}{dt} = 0,$$

find the resulting equations and state what these equations represent.

5. In quantum Mechanics the Pauli spin matrices for spin-1/2 particles (like the electron), are defined by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

Find the eigenvalues and the eigen vectors for these matrices.

6. From introductory physics you know that light travels with a speed, $c = 3 \times 10^8 m/s$, in vacuum. When it travels in a medium with a refractive index, $n > 1$ the light slows down and the speed, v is given by

$$v = \frac{c}{n}. \tag{3}$$

You also know that when light travels from one medium with refractive index $n_1$to another with refractive index, n_2, the light could bend towards or away from the normal depending on which refractive index is greater or less (Fig. 3). Using Calculus of variation show that,

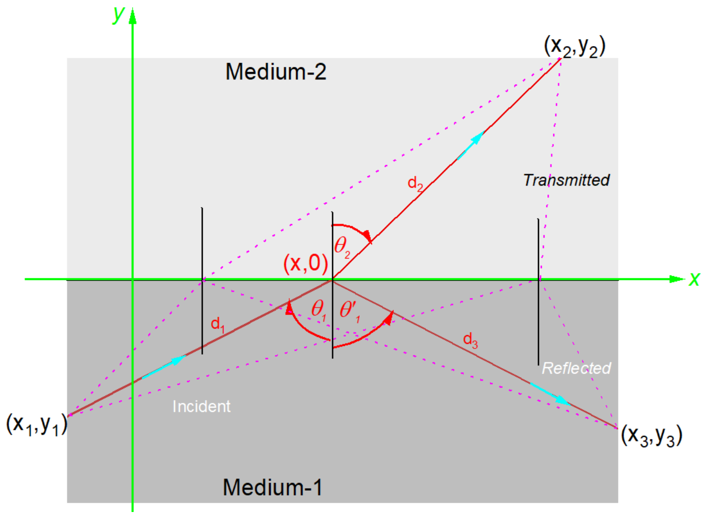

Figure 3: Different straight-line paths for the light traveling from point1 to point 2. The incident light partly gets reflected and partly gets refracted. The shortest paths are defined by the law of reflection and law of refraction.

(a) The angle of incidence, θ_1, and angle of transmission, θ_2, are related by the law of refraction (Snell's law)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \tag{4}$$

Hint:Minimize the time taken by the light

$$I = \int_1^2 dt, \tag{5}$$

Note that dt is an infinitessimal time for the light to travel an infinitessimal distance ds along the path of the light.

(b) The angle of incidence, θ_1, and the angle of reflection, θ'_1,are related by the low of reflection

$$\theta_1 = \theta'_1. \tag{6}$$

Hint:Minimize the time taken by the light

$$I = \int_1^3 dt. \tag{7}$$

N.B. In either case the light travels in a straight line.

7. The volume charge density, $\rho(\vec{r})$, of a point charge, q, placed at a point on the z-axis, $\vec{r}_0 = a\hat{z}$, can be expressed as

$$\rho(\vec{r}) = q\sigma(\vec{r} - \vec{r}_0), \tag{8}$$

where $\sigma(\vec{r})$ is the Dirac Delta function. Show that the electric potential, $V(\vec{r})$, due to this point charge is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r} - \vec{r}_0|} = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + y^2 + (z-a)^2}}. \tag{9}$$

The electric potential for a volume charge distribution is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\vec{r}')\,d\tau'}{|\vec{r} - \vec{r}'|},$$

where $\vec{r}'$ is the position of the infinitesimal charge $dq' = \rho(\vec{r}')\,d\tau'$, in an infinitesimal volume $d\tau'$, $\rho(\vec{r}')$ is the charge density in the volume V.

8. Sow that

$$\lim_{n\to\infty}\left[\frac{\Gamma\left(n+\frac{3}{2}\right)}{\sqrt{n}\Gamma\left(n+1\right)}\right]\simeq 1.$$

9. Use Gamma and Beta function formulas to show that

$$\int_0^\infty \frac{dy}{(1+y)\sqrt{y}}=\pi.$$

Equations that you may need...

- $$\Gamma\left(p+1\right) \simeq p^p e^{-p}\left(2\pi p\right)^{\frac{1}{2}}$$

- For $F\left(t, x, \dot{x}\right)$
$$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right) - \frac{\partial F}{\partial x} = 0,$$
where
$$\dot{x} = \frac{dx}{dt}.$$