PHYS 3160   Midterm Exam

Instructor: Dr. Daniel Erenso   Name:____________________________

Instructions and important notes

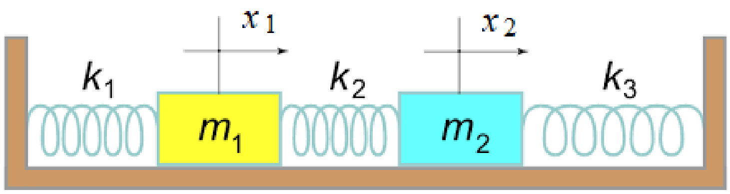
- This is the take-home part of the midterm exam. The solutions must be turned in at 12:40 on **Wednesday, March 04, 2020 during the in-class exam period.**
- Some of the problems require a specific method to solve. You must use the specific methods stated in the problem.
- Please pay attention to italicized or bold phrases.
- To receive full credit, your work must be clear and complete.
- Begin the solution of each problem on a new page. Do not use the back pages!
- The solutions to each problems must be presented in order.
- You must box in the final result to each part of the problems when it is appropriate.
- You must attach the cover page of this exam on top of the pages of your properly ordered solutions.

Problem	1	2	3	Total
Score	/20	/20	/10	/50

Part II: Take Home   Name:____________________________

Problem # 1

Imagine there is a one-dimensional chain of molecules made of two alternating different atoms with mass m_1 and m_2. The force between the atoms is assumed to be linear. This force is proportional to the displacement of the atoms from the equilibrium position. Suppose there are only two atoms in this chain as shown in the figure, we can imagine that each atom is connected to its adjacent atoms by a spring with spring constant k_2. These two atoms are interacting with the outside environment. Let's assume that the force between the atoms and the outside environment at the two ends can be described by two springs of spring constant k_1 at one end and k_3 at the other end (see Fig.).



Two masses and three springs.

The springs are at their unstretched/uncompressed lengths when the masses are at their equilibrium positions. At $t = 0$, the atoms are displaced from their equilibrium positions by the amounts x_{10} and x_{20} and then released. The speed for each mass at this time ($t = 0$) is zero.

(a) Find the kinetic energy, the potential energy, and the Lagrangian. **Using the Euler-Lagrange equation** derive the equations of motion for each masses and express the equations using matrices

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = M \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{1}$$

[4 pts]

(b) Let's assume that the two atoms have nearly the same mass (i.e. $m_1 \simeq m_2 = m$) and ,

$$k_1 = k, k_2 = 2k, k_3 = k. \tag{2}$$

Using Similarity Transformation find the Eigenvalues and Eigenvectors for the matrix, M. [4 pts]

(c) For the two masses find the displacements ($x_1(t)$ and $x_2(t)$) and speeds ($\dot{x}_1(t)$ and $\dot{x}_2(t)$). [4 pts]

(d) Find the propagator matrix. [2 pts]

(e) Describe the Normal Modes of Vibration of the atoms. [2 pts]

Problem # 2

Consider a mass, m, attached to one end of a spring with spring constant, k,sitting on a frictionless floor of a shallow swimming pool as shown in Fig. 1 The other end of the spring is attached to the wall of the pool. The mass is

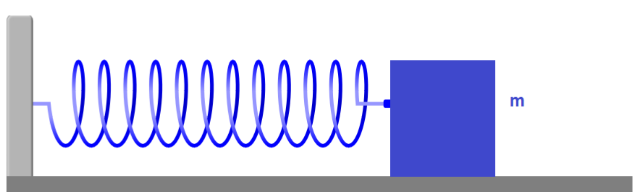


Figure 1: A harmonic oscillator in a horizontal plane.

compressed a distance x_0 and then released. The total force on the mass in the vertical direction is zero at all time.

One the other hand the net force in the horizontal direction, (x-direction) is not. The mass experiences a spring force, F_s in opposite direction to its displacement from the equilibrium position, x which is given by

$$F_s = -kx. \tag{3}$$

Furthermore due to the viscosity of the water, the mass experiences a drag force, F_d,

$$F_d = -\beta v_x. \tag{4}$$

where β is a constant that depends on the size and shape of the mass. and the viscosity of the water.

(a) Using Newton's second law show that the equation of motion for the mass is given by

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = 0. \tag{5}$$

where

$$\gamma = \frac{\beta}{2m}, \omega = \sqrt{\frac{k}{m}}$$

[2 pts]

(b) Using *any of the methods you were introduced to last semester*, find the general solution to this differential equation. [5 pts]

(c) Let's consider the case where

$$\gamma = 0, \omega = 2 \tag{6}$$

so that the differential equation becomes

$$\frac{d^2x}{dt^2} + 4x = 0. \tag{7}$$

Use *series substitution method* and find the solution to the differential equation and show that the result agrees with the result you found in part (b) when you set $\gamma = 0$. [8 pts]

Problem # 3

An electric dipole consists of two equal and opposite charges ($\pm q$) separated by a distance d. The electric field vector and lines for a dipole are very different from a monopole (see Fig. 2). We are interested in the behavior of the field

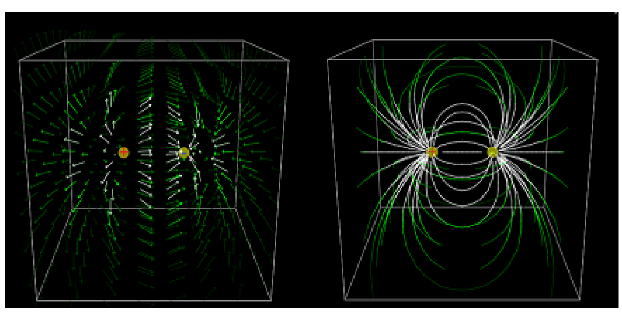


Figure 2: Electric field vector and lines of a physical dipole.

of a dipole at a point far away from the dipole which is given by

$$\vec{E}(\vec{r}) = -\nabla V(\vec{r}).$$

Use the Legendre Polynomials to find the approximate potential, $V(\vec{r})$, of a dipole at points far from the dipole (i.e.$r >> d$). See Fig. 3 [10 pts]

Note: The potential is the sum of the potential for the positive and negative charges.

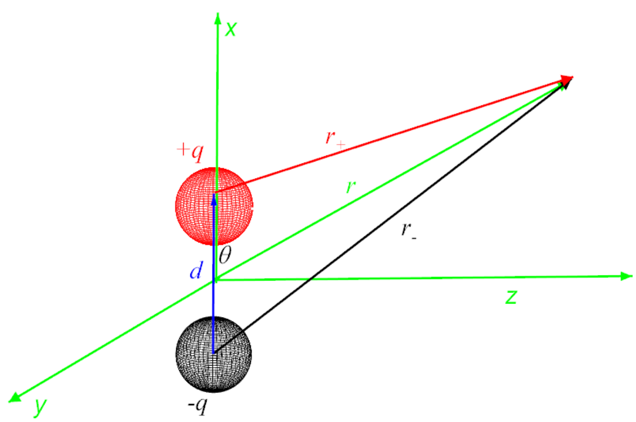


Figure 3: A dipole on the x-axis centered about the origin.