PHYS 4330 ELECTRICITY & MAGNETISM II

REVIEW HOMEWORK ASSIGNMENT 00

DUE DATE: February 04, 2020

Instructor: Dr. Daniel Erenso   Name: ______________________________

Mandatory problems: 4 (a) & (b), 5

Student signature:____

Comment:____
____
____

P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

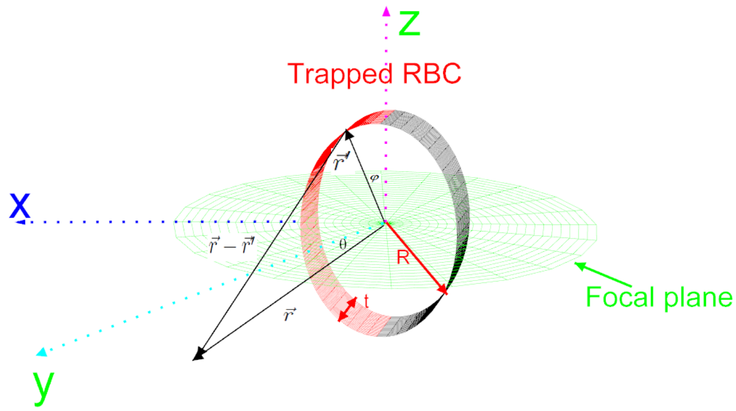


Figure 1: A simplified model for a trapped red blood cell. The cell is modeled a linear dielectric disk. When it is placed in an electric field, $\vec{E}(\vec{r})$, there will be a polarization. We assume the bound charges produced the the two sides of the cell are uniformly distributed.

Problem 1-4 Electrostatic: A ring dipole: Consider a ring with radius R siting on the x-z plane centered about the origin as shown in Fig. 1. Suppose the angle between a vector, $\vec{r}'$ describing a point on the ring and the z-axis is φ' and the angle between a vector $\vec{r}$ describing a point in space and the y-axis is θ. The projection of the vector $\vec{r}$ on the x-z plane subtends an angle φ from the z-axis. The ring carries a positive charge q on one side and a negative charge $-q$ on the other side. These charges are uniformly distributed on both sides.

1. For the set-up shown in the Fig. Noting that the Cartesian coordinate for the vector, $\vec{r}$,

$$x = r\sin(\theta)\sin(\varphi), y = r\cos(\theta), z = r\sin(\theta)\cos(\varphi) \tag{1}$$

and for the vector, $\vec{r}'$,

$$x' = R\sin(\varphi'), y' = 0, z' = R\cos(\varphi') \tag{2}$$

show that

$$|\vec{r}-\vec{r}'| = \sqrt{r^2+R^2-2rR\sin(\theta)\cos(\varphi-\varphi')} \tag{3}$$

2. Find the charge densities for the two sides of the ring.

3. Show that the potential at a point in space described by the vector $\vec{r}$ can be expressed as

$$V(\vec{r}) = \frac{q}{4\pi^2\epsilon_0\sqrt{r^2+R^2}}\left[\int_0^\pi \frac{d\varphi'}{\sqrt{1-u\cos(\varphi-\varphi')}} - \int_0^\pi \frac{d\phi}{\sqrt{1+u\cos(\varphi-\phi)}}\right]. \tag{4}$$

where

$$u = \frac{2rR\sin(\theta)}{r^2+R^2} \tag{5}$$

4. Using series expansion show

(a) that the potential in (3) can be put in the form

$$V(\vec{r}) = \frac{q}{4\pi^2\epsilon_0\sqrt{r^2+R^2}}\sum_{n=0}^{\infty}\begin{pmatrix} -1/2 \\ n \end{pmatrix} u^n \int_0^\pi \left((-1)^n - 1\right)\cos^n(\varphi-\varphi')\, d\varphi' \tag{6}$$

where

$$\begin{pmatrix} -1/2 \\ n \end{pmatrix} = \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}-3\right)...\left(-\frac{1}{2}-n+1\right)}{n!} \tag{7}$$

(b) Show that the potential in (a) near the axis where $\sin(\theta) << 1$ the potential can be approximated as

$$V(\vec{r}) \simeq -\frac{qR}{\pi^2\epsilon_0}\frac{r\sin(\theta)\sin(\varphi)}{(r^2+R^2)^{3/2}}. \tag{8}$$

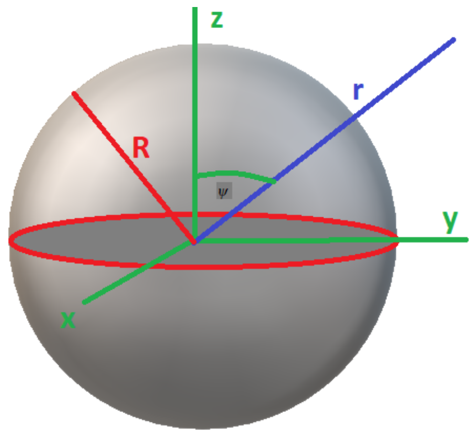
(c) Express the result in (b) using Cartesian coordinates

(d) Find the approximate relation for the potential near the center of the ring.

(e) Find the electric field near the axis of the ring.

5. *Magnetostatic*

(a) A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω about the z-axis. Find the vector potential and the magnetic field both inside and outside the sphere.



Hint: Rotate the position $\vec{r}$ where we want to determine the vector potential by an angle ψ in a counter clockwise direction so that it coincides with the positive $z-$axis as shown in the figure below.

(b) Applying the result in mart (a), find the magnetic field of a uniformly magnetized sphere of radius R and magnetization, $\vec{M} = M\hat{z}$.

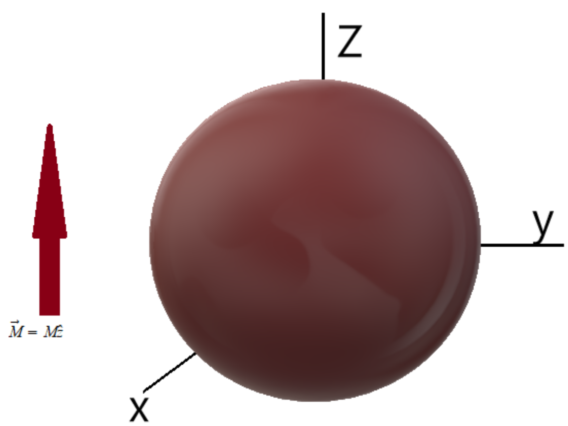


Figure 2: Uniformly magnetized sphere.

Hint: Discuss what a uniform magnetization along the z direction mean in terms of the bound current density.