## Instructions and important notes

- This is the take-home part of the midterm exam. The solutions must be turned in at 09:40 on Thursday, March 05, 2020 during the in-class exam period.
- Some of the problems require a specific method to solve. You must use the specific methods stated in the problem.
- Please pay attention to italicized or bold phrases.
- To receive full credit, your work must be clear and complete.
- Begin the solution of each problem on a new page. Do not use the back pages!
- The solutions to each problems must be presented in order.
- You must box in the final result to each part of the problems when it is appropriate.
- You must attach the cover page of this exam on top of the pages of your properly ordered solutions.

| Problem | 1 | 2 | 3 | Total |
| :---: | :---: | :---: | :---: | :---: |
| Score | $/ 15$ | $/ 20$ | $/ 15$ | $/ 50$ |

## Take Home Part

## Name:

## Problem \# 1

A line charge is glued onto the rim of a wheel of radius, $b$, which is then suspended horizontally, as shown in Fig. 1, so that it is free to rotate (the spokes are made of some nonconducting material). In the central region, out to radius $a$, there is a magnetic field pointing along the positive z-axis $\left(\vec{B}=B_{0} \hat{z}\right)$. This magnetic field begins to decrease at a rate of $\alpha$,

$$
\frac{d B}{d t}=-\alpha
$$



Figure 1: A circular (radius $b$ ) line charge (pink) glued to none conducting wheel (on the x-y plane) connected by a none conducting spokes (green) is free to rotate about an axle. A uniform magnetic field along the z-direction confined to a region with radius, $a$ (red) is decreasing at a rate of $\alpha$.
(a) Explain qualitatively and quantitatively what is going to happen and why is it happening from Faraday's law [5 pts]
(b) Show that the angular momentum imparted to the wheel does not depend on $\alpha$. [5 pts]
(c) Explain qualitatively what is going to happen and why is it happening from angular momentum in electromagnetism. You must be able to explain in terms of the momentum carried by the fields.[ 5 pts ]

## Problem \# 2

A long coaxial cables of length $l$ and a spherical capacitor are connected in series to a dc power supply as shown in Fig. 2. At a given instant of time the current, $I(t)$, flows down the inner solid cylinder, radius $a$, and back along the surface of the outer cylindrical shell, radius $b$. The inner cylinder has a magnetic permeability, $\mu_{0}$ The space between the inner cylinder and outer cylinder is filled by a material with magnetic and electrical susceptibilities, $\mu$ and $\epsilon$, respectively except a tiny wire shown in the figure for the purpose of current flow from the inner to the outer. The current from the outer cylinder then flows through a wire to the outer conducting spherical shell of radius, $b$ The inner sphere with radius $a$, which is insulated (by vacuum) from the outer spherical shell is connected to the negative terminal of the dc power supply. Let the resistance of the conducting material that made up the cables, the wires, and the capacitor be $R$. and the dc power source voltage be $V_{0}$. Note that $l \gg a, b$ for the coaxial cables.
(a) Find the inductance and capacitance per unit length for the coaxial cables. Find the product of these two quantities and discuss its physical significance? [5 pts]
(b) Find the capacitance for the spherical capacitor that serve as a capacitor in the circuit.[5 pts]
(c) Assuming initially the capacitor is uncharged $(Q(0)=0)$ and there is no current flow $(I(0)=0)$ in the circuit find the charge on the capacitor and the current in the circuit as a function of time. [5 pts]
N.B. You can use $L$ for the inductance and $C$ for the capacitance when you solve this part. Don't use the explicit forms of $L$ and $C$ you determined in part (a) and (b).


Figure 2: A spherical capacitor and two coaxial cylinders connected to a dc power supply.
(d) Show that the rate of energy supplied by the power source is the same as the rate of energy consumed by the element of the circuit (capacitor, inductor, and resistor) [5 pts]
NB: Your work must show how you determined the fields when it is necessary to use the fields in the question being asked.

## Problem \# 3

A long straight conductor carrying a current, $I_{1}$, and ring of radius, $a$, carrying a current, $I_{2}$, lie in the same plane as shown in Fig. 3 (i.e. y-z plane). The distance between the wire and the center of the ring is $b$.
(a) Find the mutual inductance $M$ [7 pts]
(b) Find the force $F$ between the two conductors. [8 pts]


Figure 3: A long wire and a circular loop on the $y-z$ plane.

NB: You can refer to table of integrals or use mathematica whenever its becomes difficult to integrate.

