PHYS 4390 Quantum Mechanics II

Homework Assignment 03

Due date: February 12, 2019

Instructor: Dr. Daniel Erenso Name: ______________________________

Mandatory problems: *any two problems is required but I want you to try all!*

Student signature:______________________________________

Student Comment:__________________________________

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P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

Prob 1 Show that the

$$\left[\left(\hat{p}^2\right)^2, \hat{L}\right] = 0, \left[\hat{S}\cdot\hat{L}, \hat{J}_z\right] = 0$$

where

$$\hat{J}_z = \hat{L}_z + \hat{S}_z$$

Prob 2 Show that

$$2\left(\hat{S}\cdot\hat{L}\right) = \hat{L}_+\hat{S}_- + \hat{L}_-\hat{S}_+ + 2\hat{L}_z\hat{S}_z$$

where $\hat{S}$ is spin and $\hat{L}$ is orbital angular momentum operators, $\hat{L}_z$ and $\hat{S}_z$ are the corresponding z-component operators satisfying the eigenvalue equation

$$\hat{L}_z\left|l,m\right\rangle = \hbar m\left|l,m\right\rangle, \hat{S}_z\left|\frac{1}{2},\pm\frac{1}{2}\right\rangle = \pm\frac{\hbar}{2}\left|\frac{1}{2},\pm\frac{1}{2}\right\rangle.$$

The raising ($+$) and lowering ($-$) operators given by

$$\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y, \hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y,$$

satisfy the equation

$$\begin{aligned}\hat{L}_\pm\left|l,m\right\rangle &= \hbar\sqrt{\left(l(l+1) - m\left(m\pm1\right)\right)}\left|l,m\pm1\right\rangle,\\ \hat{S}_\pm\left|l_s,m_s\right\rangle &= \hbar\sqrt{\left(l_s(l_s+1) - m_s\left(m_s\pm1\right)\right)}\left|l_s,m_s\pm1\right\rangle.\end{aligned}$$

Prob 3 Following a similar procedure we followed in class show that

$$\begin{aligned}\left\langle n,l,m,\chi_1\right|\hat{H}_1\left|n,l,m,\chi_1\right\rangle &= \hbar^2 mF\left(r\right),\\ \left\langle n,l,m+1,\chi_2\right|\hat{H}_1\left|n,l,m+1,\chi_2\right\rangle &= -\hbar^2\left(m+1\right)F\left(r\right)\end{aligned}$$

where

$$F\left(r\right) = \frac{Ze^2}{8\pi\epsilon_0 m_e^2c^2}\frac{Z^3}{a_0^3n^3l\left(l+\frac{1}{2}\right)\left(l+1\right)}$$

and

$$\hat{H}_1 = \frac{Ze^2}{8\pi\epsilon_0 m_e^2c^2}\frac{2\left(\hat{S}\cdot\hat{L}\right)}{r^3} = \frac{Ze^2}{8\pi\epsilon_0 m_e^2c^2}\frac{1}{r^3}\left(\hat{L}_+\hat{S}_- + \hat{L}_-\hat{S}_+ + 2\hat{L}_z\hat{S}_z\right),$$

$$\left|\chi_1\right\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \left|\chi_2\right\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

Prob 4 Determine the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} m & \sqrt{l\left(l+1\right) - m\left(m+1\right)} \\ \sqrt{l\left(l+1\right) - m\left(m+1\right)} & -(m+1)\end{bmatrix}$$

Prob 5 Neglecting the intrinsic spin of the electron, the Hamiltonian for an electron in a hydrogen atom subject to a constant magnetic field $\vec{B}$ is given by

$$\hat{H} = \frac{\hat{p}^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0 r} + \frac{e}{2m}\widehat{\vec{L}}\cdot\widehat{\vec{B}}$$

where $\vec{L}$ is the orbital angular momentum operator. In the absence of the magnetic field, there will be a single line in the transition from an $(n=4, l=3)$ state to an $(n=3, l=2)$ state. What will be the effect of the magnetic field on that line? Sketch the new spectrum and the possible transitions, constrained by the selection rule (i.e. $\Delta m = 0, \pm1$). How many lines will there be? What will be the effect of a constant electric field $\vec{E}$ parallel to $\vec{B}$.