PHYS 4390 Quantum Mechanics II

Homework Assignment 04

Due date: February 19, 2019

Instructor: Dr. Daniel Erenso Name: ______________________

Mandatory problems: *any two problems is required but I want you to try all!*

Student signature:_______________________________________

Student Comment:____________________________________
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P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

Prob 1 Find all the energy values for the ground and the first excited state for a hydrogen atom in a uniform magnetic field (apply Eq. (11.300) in my note) with no relativistic correction included and draw the energy spectrum.

Prob 2 The quantum Hamiltonian for a Hydrogenic atom with spin-orbit coupling correction in a uniform magnetic field directed is given by

$$\hat{H} = \frac{\hat{p}^2}{2m_e} - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} + \frac{Ze^2}{4\pi\epsilon_0 m_e^2 c^2}\frac{\vec{S}\cdot\vec{L}}{r^3} + \frac{e}{2m_e}\left(\vec{J}+\vec{S}\right)\cdot\vec{B}.$$

Show that:

(a) The total angular momentum operator

$$\hat{J} = \vec{L} + \vec{S},$$

commutes with the Hamiltonian $\hat{H}$

(b)

$$\left[\vec{L},\hat{H}\right] \neq 0, \left[\vec{S},\hat{H}\right] \neq 0.$$

Prob 3 The correction to the quantum Hamiltonian for a Hydrogenic atom due to the magnetic interaction of the Nucleus with the electron is given by

$$\hat{H}_N = \frac{Ze^2}{4\pi\epsilon_0}\frac{g_N}{4m_e m_N c^2}\left[\frac{3\hat{r}\left(\hat{r}\cdot\hat{S}_N\right) - \hat{S}_N}{r^3} + \frac{8\pi}{3}\hat{S}_N\delta\left(\vec{r}\right)\right]\cdot\left(\hat{L}+2\hat{S}\right).$$

(a) Using the unperturbed states for a Hydrogenic atom (with no spin-orbit coupling included) show that the energy correction for the ground state

$$\Delta E_N = \langle 1,0,0,\chi_k|\,\hat{H}_N\,|1,0,0,\chi_k\rangle = \frac{2}{3}\left(Z\alpha\right)^4 m_e c^2 g_N \frac{m_e}{m_N}\left(F\left(F+1\right) - S\left(S+1\right) - S_N\left(S_N+1\right)\right)$$

where

$$F = S + S_N$$

is the total spin quantum number for the nucleus and the electron. Find the numerical value for the ground state of the Hydrogen atom.

(b) Determine

$$\Delta E_N = \langle n,l,m,\chi_k|\,\hat{H}_N\,|n,l,m,\chi_k\rangle$$

for the first excited state.

Prob 4 Show that

$$\Psi\left(\vec{r},\vec{R}\right) = e^{\frac{i\vec{P}\cdot\vec{R}}{\hbar}}u\left(\vec{r}\right)$$

is the solution to the equation

$$\left[\frac{\hat{p}^2}{2\mu} + \frac{\hat{P}^2}{2M} + V\left(\vec{r},\vec{R}\right)\right]\Psi\left(\vec{r},\vec{R}\right) = E\Psi\left(\vec{r},\vec{R}\right).$$

where $\vec{P}$ is the eigenvalue of $\hat{P}$ and $u\left(r\right)$ satisfies the equation

$$\left[\frac{\hat{p}^2}{2\mu} + V\left(\vec{r}\right)\right]u\left(r\right) = \left(E - \frac{P^2}{2M}\right)u\left(r\right)$$

that we obtain after we do the separation of variables.

Prob 5 Suppose we replace photon 1 by electron 1 and photon 2 by electron 2 with the same momentum. We use spin instead of polarization for electrons so let's say electron 1 has spin up $|+z\rangle$ and electron 2 has spin down $|-z\rangle$. Suppose the 50/50 polarizing beam splitter is replaced by a similar device capable of interchanging both the momentum and spin of the electrons. Write at least five states like in Example 12.1 and show that which of these states are possible two electron state.