PHYS 4800 HOMEWORK 01

DUE DATE January 31, 2018

Instructor: Dr. Daniel Erenso Name: ______________________________

Declaration:
It am expected to solve all the five problems assigned for this homework set to get a full credit. I have tried all my best to solve all the five problems. I have submitted the solutions of _ _ _ _ _ _ _ _ _ _Problems. All the solutions are solely the result of my own work. I am also fully aware that only two problems selected by Dr. Erenso will be graded and scored according to the outline given in syllabus.
Signature:_ _

P #	1	2	3	4	5	Score
Score	/	/	/	/	/	/100

1. You heard in the News that there are two events happened somewhere in this planet. Suppose event one occurred at time t_1 at a point in space (x_1, y_1, z_1), which we may describe using spacetime coordinates (t_1, x_1, y_1, z_1), as recorded by an observer on an inertial reference frame S. The second event occurred at a later time t_2 at another point in space (x_2, y_2, z_2) as recorded by the same observer. Show that the time difference

$$\Delta t = t_2 - t_1 \tag{1}$$

and the quantity

$$(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \tag{2}$$

are, separately, invariant under any Galilean transformation. Note that

$$\Delta x = x_2 - x_1, \Delta y = y_2 - y_1, \Delta z = z_2 - z_1 \tag{3}$$

You must show that

$$\Delta t' = \Delta t, (\Delta r')^2 = (\Delta r)^2$$

2. Consider the two events in problem 1 described by the spacetime coordinates (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2). Show that the *interval* between these two events squared

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2, \tag{4}$$

is invariant under the Lorentz transformation.

3. Using the Lorentz transformation

$$\begin{bmatrix} ct \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cosh(\psi) & -\sinh(\psi) & 0 & 0 \\ -\sinh(\psi) & \cosh(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}. \tag{5}$$

Show that the interval squared between the two events in problem 1 is invariant.

4. Suppose the couples on the spacecraft celebrated their child (a girl) sweet sixteen birthday as measured by a clock on board the spacecraft (S'). The girl is about $1.6m$ tall as measured by her parents. Assume the spacecraft is traveling with constant velocity $v = 0.8c$, where c is the speed of light in vacuum.

(a) What would be the age of the girl as measured by an observer on earth (S inertial frame).

(b) How tall is the girl as measured by an observer on earth, (S inertial frame).

5. Consider three inertial reference frames S, S', and S''. Suppose S' is related to S by a boost of speed v in the x direction and that S'' is related to S' by a boost of speed u' in the x'-direction. Using the rapidity parameter defined as

$$\psi_v = \tanh^{-1}\left(\frac{v}{c}\right), \psi_{u'} = \tanh^{-1}\left(\frac{u'}{c}\right). \tag{6}$$

show that

(a)

$$\begin{aligned} ct'' &= ct\cosh(\psi_v + \psi_{u'}) - x\sinh(\psi_v + \psi_{u'}), \\ x' &= -ct\sinh(\psi_v + \psi_{u'}) + x\cosh(\psi_v + \psi_{u'}), \\ y' &= y, \\ z' &= z. \end{aligned}$$

.

(b)

$$u = ct\tanh(\psi_v + \psi_{u'}) = c\frac{\tanh\psi_v + \tanh\psi_{u'}}{1 + \tanh\psi_v \tanh\psi_{u'}} = \frac{u' + v}{1 + u'v/c^2}$$