PHYS 4800 HOMEWORK 04

DUE DATE: Before spring break

Instructor: Dr. Daniel Erenso Name: ——————————————

Declaration:
It am expected to solve all the five problems assigned for this homework set to get a full credit. I have tried all my best to solve all the five problems. I have submitted the solutions of __________Problems. All the solutions are solely the result of my own work. I am also fully aware that only two problems selected by Dr. Erenso will be graded and scored according to the outline given in syllabus.
Signature:__________________________________

P #	1	2	3	4	5	Score	F. Score
Score	/	/	/	/	/	/100	/100

Prob 1 *Consider the 3D sphere of radius, a, in a 4D Euclidean space. A point P on this 3D sphere is described by the vector*

$$\vec{s} = r\sin(\theta)\cos(\varphi)\,\hat{x} + r\sin(\theta)\sin(\varphi)\,\hat{y} + r\cos(\theta)\,\hat{z} + \sqrt{a^2 - r^2}\hat{w}$$

(a) Find the basis vectors $\hat{e}_r, \hat{e}_\theta$, and $\hat{e}_\varphi$ in the tangent space at point P.

(b) Re-derive the metric elements for a 3D sphere from the basis vectors.

Prob 2 Problem 2.10 from the text

Prob 3 For the a coordinate transformation $x^a \to x'^a$, show that at a point on a maniforld, the dual basis vector are transformed by the equation

$$\hat{e}'^a = \frac{\partial x'^a}{\partial x^c}\hat{e}^c.$$

Prob 4 For the a coordinate transformation $x^a \to x'^a$, show that at a point on a maniforld the covariant components of a vector $\vec{v}$ transformed by the equation

$$v'_b = \frac{\partial x^a}{\partial x'^b}v_a.$$

Prob 5 Problem 2.13 from the text

Prob 6 *Optional:* This is a problem requires your understanding of the special theory of relativity we covered in the first chapter and E & M:

Consider a long conducting wire carrying a current I along the x-axis on the S frame. This current can be visualized as a string of positive charge described by a charge density λ moving along the positive x-direction with velocity v measured in the S frame. Superimposed on the positive charge there is a negative charge with the same charge density λ moving in the negative x-direction with the same magnitude of velocity. The the current generated will then be

$$I = 2\lambda v$$

and the magnetic field at a distance a from the line charge

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 \lambda v}{\pi a}$$

Now consider a point charge q moving along the positive x-direction with a velocity $u < v$ at a distance a from the line charge. For an observer on the S frame where we measured the velocities (both u and v), since the total charge is zero the electrical force on the point charge is zero and the charge experiences only magnetic force given by

$$F_m = -quB = -\frac{\mu_0 q u \lambda v}{\pi s}$$

For an observer on the S' frame moving with a velocity v in the positive x-direction, show that the force is actually an electrical force given by

$$F'_\perp = F'_{el} = -\frac{\mu_0 q \lambda u v}{\pi s}\frac{1}{\sqrt{1 - u^2/c^2}}.$$

Hint: You need length the length and velocity transformations. If you can show that without looking at E & M text book or my E & M note you must be proud of yourself!!! You have master E & M and Special theory of relativity!