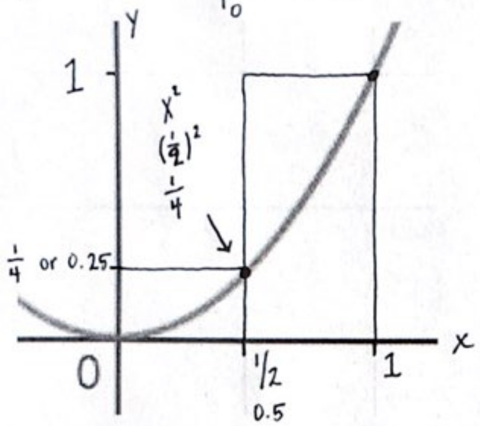
Example 1: Integration for $y = x^2$ from x = 0 to x = 1.

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Bigg|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \boxed{\frac{1}{3}\,or\ 0.333\ldots\ square\ units}$$

← Accurate area under the curve



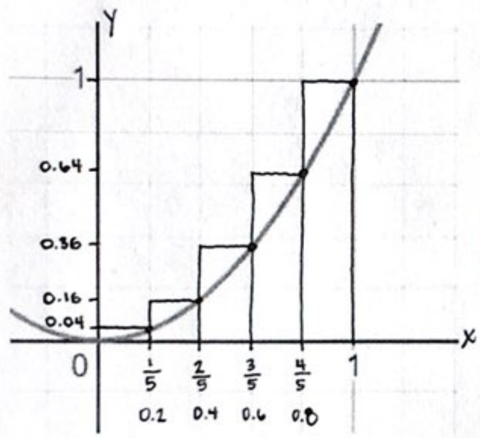
1. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (____ * ____) + (____ * ____)

   A = (__________) + (__________)

   A = ______________________________ square units
3. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...



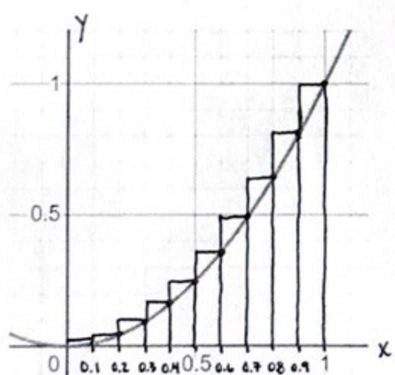
4. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)

   A = (______) + (______) + (______) + (______) + (______)

   A = ______________________________ square units
6. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...



7. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
8. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * .01) + (__ * .04) + (__ * .09) + (__ * .16) + (__ * .25)

   + (__ * .36) + (__ * .49) + (__ * .64) + (__ * .81) + (__ * 1)

   A = (__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)

   A = ______________________________ square units
9. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...

Answer these questions...

10. Which rectangle method's approximate area under the curve was closest to the accurate area? (Circle one.)

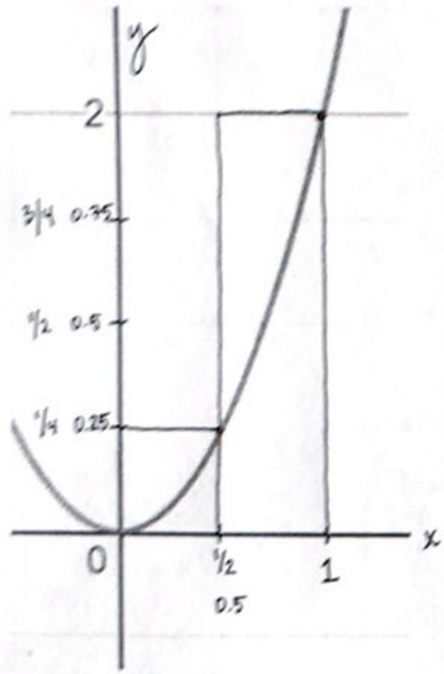
    Rectangles with a partition of ½   Rectangles with a partition of 1/5   Rectangles with a partition of 1/10

11. All of the area approximations were greater than the accurate answer (1/3 or 0.333...). Why do you think that is?

Example 2: Integration for $y = 2x^2$ from $x = 0$ to $x = 1$.

$$\int_0^1 2x^2\,dx = \frac{2x^3}{3}\Big|_0^1 = \frac{2(1^3)}{3} - \frac{2(0^3)}{3} = \frac{2}{3} - 0 = \frac{2}{3} \textit{ or } 0.666\ldots \textit{ square units}$$

← Accurate area under the curve



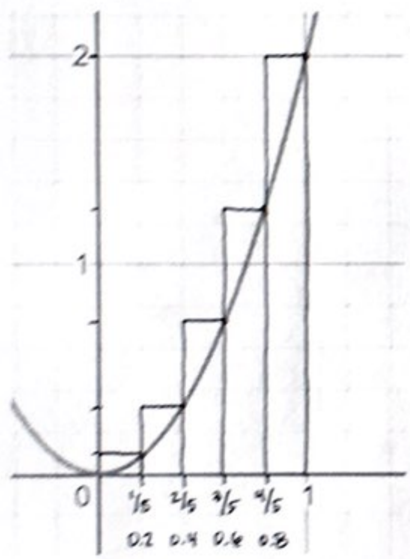
1. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (____ * ____) + (____ * ____)

   A = (__________) + (__________)

   A = ______________________________ square units
3. Is the area approximation greater or less than 2/3 or 0.666...? (Circle one.)

   Greater than 2/3 or 0.666... Less than 2/3 or 0.666...



4. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)

   A = (______) + (______) + (______) + (______) + (______)

   A = ______________________________ square units
6. Is the area approximation greater or less than 2/3 or 0.666...? (Circle one.)

   Greater than 2/3 or 0.666... Less than 2/3 or 0.666...

Answer these questions...

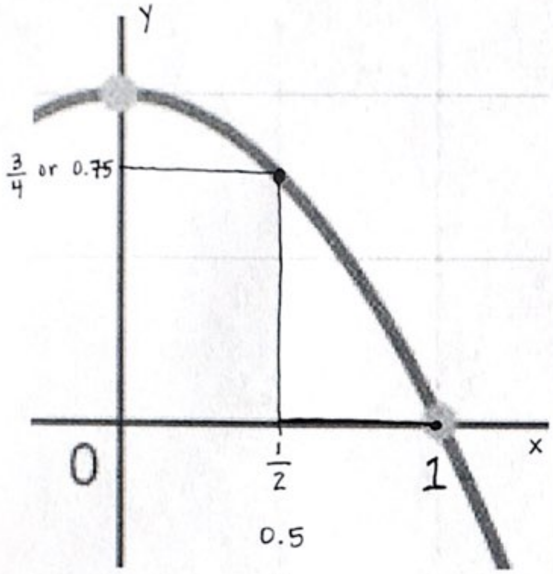
7. How does the size of the partition change the accuracy of the approximation of the area under the curve? Why?

8. What similarities and differences do you notice between the "triangle method" and the "rectangle method"?

Evaluate: Complete Example 3 on your own. You can do it!

Example 3: Integration for $y = -x^2 + 1$ from x = 0 to x = 1.

$$\int_0^1 -x^2 + 1\,dx = \left.\frac{-x^3}{3} + x\right|_0^1 = \left(\frac{-(1^3)}{3} + 1\right) - \left(\frac{-(0^3)}{3} + 0\right) = \frac{-1}{3} + 1 = \boxed{\frac{2}{3}\,or\ 0.666\ldots\,square\ units}$$



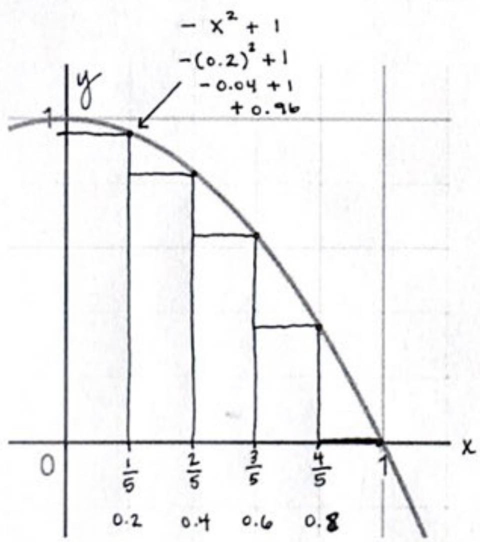
1. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (____ * ____) + (____ * <u>O</u>)

   A = (__________) + (<u>O</u>)

   A = __________________________ square units

3. Notice that the approximation is less than 2/3 or 0.666.... Why do you think this approximation is less than the accurate answer?



4. What is the length of the base of each rectangle?
   ________ (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * <u>0.96</u>) + (__ * __) + (__ * __) + (__ * __) + (__ * <u>O</u>)

   A = (______) + (______) + (______) + (______) + (______)

   A = __________________________ square units

6. Which approximation (the approximation above with a partition of ½ or this approximation with a partition of 1/5) is closer to the accurate answer? Why?

Answer this question...

7. What did you learn today in your own words? Give as much detail as you can.