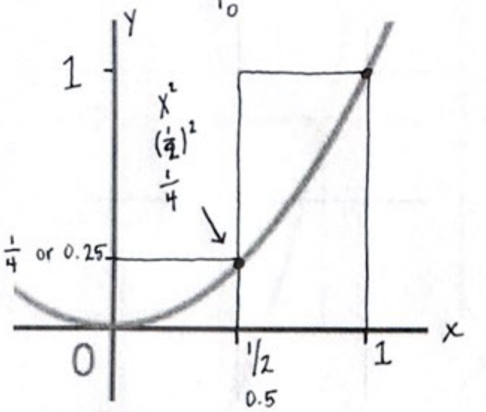
Example 1: Integration for $y = x^2$ from x = 0 to x = 1.

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Bigg|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \boxed{\frac{1}{3} \text{ or } 0.333\ldots \text{ square units}}$$

Accurate area under the curve



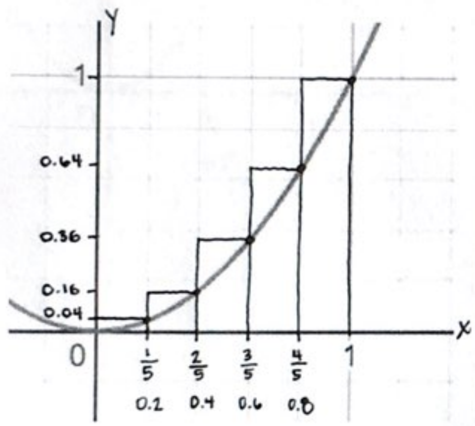
1. What is the length of the base of each rectangle?
   ½ or 0.5 (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (0.5 * 0.25) + (0.5 * 1)

   A = (0.125) + (0.5)

   A = 0.625 square units
3. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...



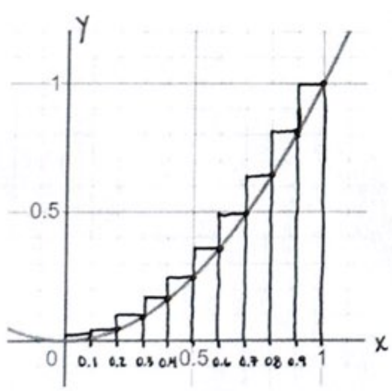
4. What is the length of the base of each rectangle?
   ⅕ or 0.2 (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (0.2 * .04) + (.2 * .16) + (.2 * .36) + (.2 * .64) + (.2 * 1)

   A = (.008) + (.032) + (.072) + (.128) + (0.2)

   A = 0.44 square units
6. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...



7. What is the length of the base of each rectangle?
   1/10 or 0.1 (<- in calculus, this is called a partition)
8. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (.1 * .01) + (.1 * .04) + (.1 * .09) + (.1 * .16) + (.1 * .25)

   + (.1 * .36) + (.1 * .49) + (.1 * .64) + (.1 * .81) + (.1 * 1)

   A = (.001)+(.004)+(.009)+(.016)+(.025)+(.036)+(.049)+(.064)+(.081)+(.1)

   A = 0.385 square units
9. Is the area approximation greater or less than 1/3 or 0.333...? (Circle one.)

   Greater than 1/3 or 0.333...   Less than 1/3 or 0.333...

Answer these questions...

10. Which rectangle method's approximate area under the curve was closest to the accurate area? (Circle one.)

    Rectangles with a partition of ½   Rectangles with a partition of 1/5   Rectangles with a partition of 1/10

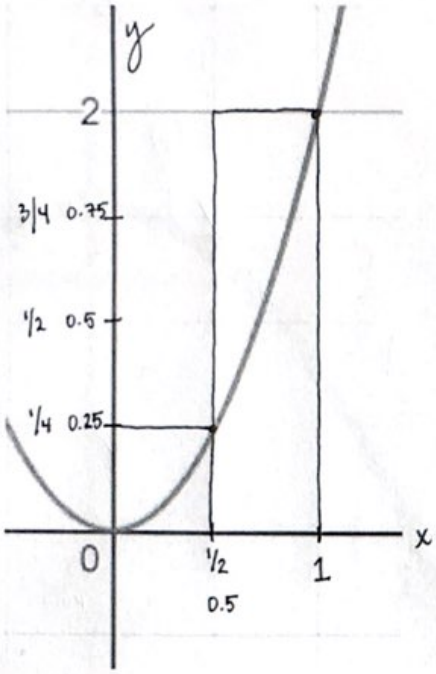
11. All of the area approximations were greater than the accurate answer (1/3 or 0.333...). Why do you think that is?

    The rectangles overestimate the area under the curve because they cover area outside of the boundary of the curve.

Example 2: Integration for $y = 2x^2$ from x = 0 to x = 1.

$$\int_0^1 2x^2\,dx = \frac{2x^3}{3}\Bigg|_0^1 = \frac{2(1^3)}{3} - \frac{2(0^3)}{3} = \frac{2}{3} - 0 = \boxed{\frac{2}{3} \text{ or } 0.666\ldots \textit{ square units}}$$

Accurate area under the curve



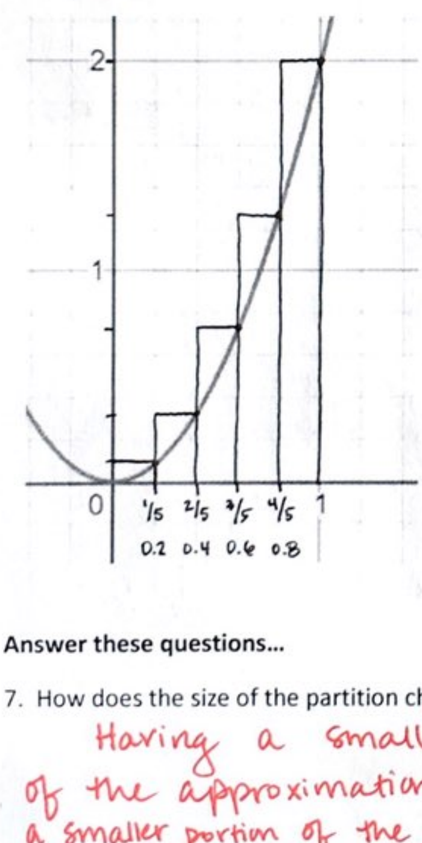
1. What is the length of the base of each rectangle?
   1/2 or 0.5 (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (0.5 * .25) + (0.5 * 2)

   A = (0.125) + (1)

   A = 1.125 square units
3. Is the area approximation greater or less than 2/3 or 0.666...? (Circle one.)

   Greater than 2/3 or 0.666... Less than 2/3 or 0.666...



4. What is the length of the base of each rectangle?
   1/5 or 0.2 (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (.2 * .08) + (.2 * .32) + (.2 * .72) + (.2 * 1.28) + (.2 * 2)

   A = (.016) + (.064) + (.144) + (.256) + (0.4)

   A = 0.88 square units
6. Is the area approximation greater or less than 2/3 or 0.666...? (Circle one.)

   Greater than 2/3 or 0.666... Less than 2/3 or 0.666...

Answer these questions...

7. How does the size of the partition change the accuracy of the approximation of the area under the curve? Why?

   Having a smaller partition size increases the accuracy of the approximation of the area under the curve because a smaller portion of the rectangles' areas extend beyond the curve's boundary.

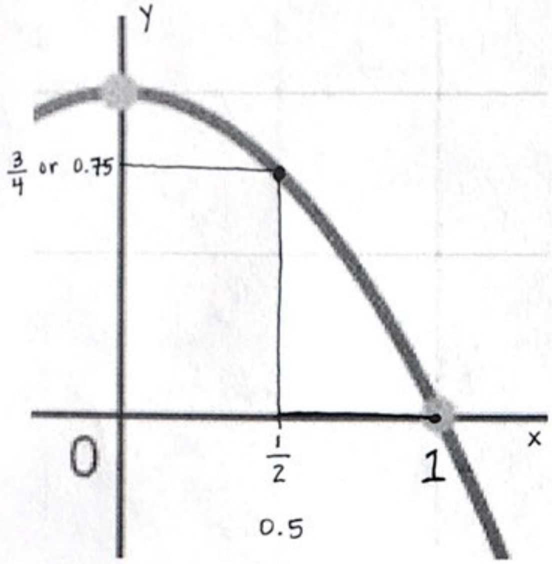
8. What similarities and differences do you notice between the "triangle method" and the "rectangle method"?

   Both use the area of 2D shapes to estimate the area under the curve. The rectangle method can be used with curves of various shapes... it is more versatile.

Evaluate: Complete Example 3 on your own. You can do it!

Example 3: Integration for $y = -x^2 + 1$ from x = 0 to x = 1.

$$\int_0^1 -x^2 + 1\,dx \;=\; \frac{-x^3}{3} + x\Big|_0^1 = \left(\frac{-(1^3)}{3} + 1\right) - \left(\frac{-(0^3)}{3} + 0\right) \;=\; \frac{-1}{3} + 1 \;=\; \boxed{\frac{2}{3}\,or\ 0.666\ldots square\ units}$$



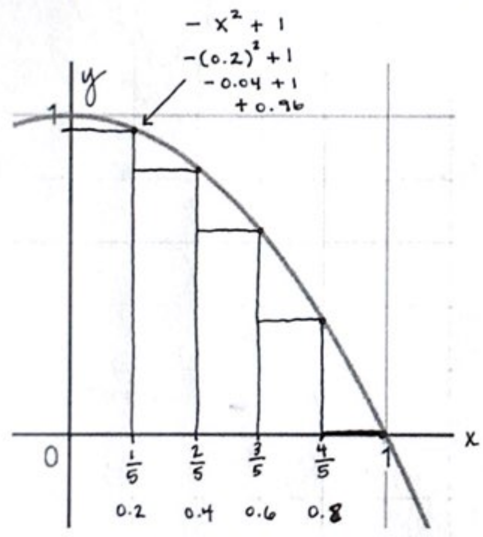
1. What is the length of the base of each rectangle?
   1/2 or 0.5 (<- in calculus, this is called a partition)
2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (0.5 * .75) + (0.5 * 0)

   A = (0.375) + (0)

   A = 0.375 square units

3. Notice that the approximation is less than 2/3 or 0.666…. Why do you think this approximation is less than the accurate answer? The rectangle is enclosed in the area under the curve. It underestimates the area under the curve.



4. What is the length of the base of each rectangle?
   1/5 or .2 (<- in calculus, this is called a partition)
5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (.2 * 0.96) + (.2 * .84) + (.2 * .64) + (.2 * .36) + (.2 * 0)

   A = (.192) + (.168) + (.128) + (.072) + (0)

   A = 0.56 square units

6. Which approximation (the approximation above with a partition of ½ or this approximation with a partition of 1/5) is closer to the accurate answer? Why?
   This approximation w/ a partition of 1/5 is closer to the accurate answer because the enclosed rectangles cover more of the area under the curve.

Answer this question…

7. What did you learn today in your own words? Give as much detail as you can.

Will vary from student to student, of course. :)