PHYS 4800 HOMEWORK 02

DUE DATE: February 12

Instructor: Dr. Daniel Erenso Name: _____

Declaration:
It am expected to solve all the five problems assigned for this homework set to get a full credit. I have tried all my best to solve all the five problems. I have submitted the solutions of _____Problems. All the solutions are solely the result of my own work. I am also fully aware that only two problems selected by Dr. Erenso will be graded and scored according to the outline given in syllabus.
Signature:_____

P #	1	2	3	4	5	Score	F. Score
Score	/	/	/	/	/	/100	/100

Prob 1 Problem # 1.5

Sol:

Prob 2 Problem # 1.3

Sol: Let's consider the two aliens instead of the particles we considered in class shown in the figure belowLet the

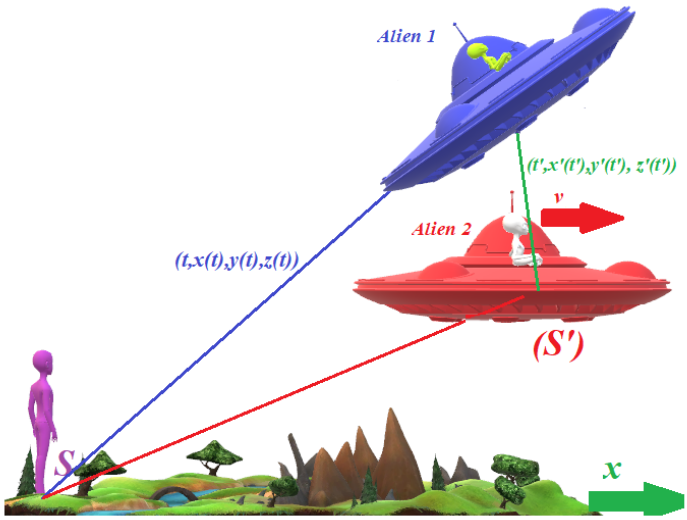

Figure 1: Two Alien spaceship traveling with different velocity relative to an observer on earth.

velocity of Alien 1 be along the y-direction normal to the direction of the second alien which is in the positive x-direction. Both magnitude of the velocity for both Aliens is the same. Then using Lorentz transformation one can write for Alien 1 relative to Aline 2

$$\begin{aligned} ct' &= \gamma\left(ct-\beta x\right), \\ x' &= \gamma\left(x-c\beta t\right), \\ y' &= y, \\ z' &= z. \end{aligned}$$

Then one can write

$$dt' = \gamma\left(dt-\frac{\beta}{c}dx\right), dx' = \gamma\left(dx-c\beta dt\right), dy' = dy, dz' = dz$$

the components of the velocity of Alien 1 relative to Alien 2

$$u'_x = \frac{dx'}{dt'}, u'_y = \frac{dy'}{dt'}, u'_z = \frac{dz'}{dt'}$$

becomes

$$\begin{aligned} u'_x &= \frac{\gamma\left(dx-c\beta dt\right)}{\gamma\left(dt-\frac{\beta}{c}dx\right)}, u'_y = \frac{dy}{\gamma\left(dt-\frac{\beta}{c}dx\right)}, u'_z = \frac{dz}{\gamma\left(dt-\frac{\beta}{c}dx\right)} \\ \Rightarrow\ u'_x &= \frac{\frac{dx}{dt}-c\beta}{1-\frac{\beta}{c}\frac{dx}{dt}}, u'_y = \frac{\frac{dy}{dt}}{\gamma\left(1-\frac{\beta}{c}\frac{dy}{dt}\right)}, u'_z = \frac{\frac{dz}{dt}}{1-\frac{\beta}{c}\frac{dz}{dt}} \end{aligned}$$

Noting that for Alien 1

$$\frac{dx}{dt} = \frac{dz}{dt} = 0, \frac{dy}{dt} = v$$

one finds

$$u'_x = -c\beta, u'_y = \frac{v}{\gamma\left(1-\frac{\beta}{c}v\right)}, u'_z = 0$$

and replacing $\beta = v/c$ and $\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$

$$u'_x = -v, u'_y = v\sqrt{1-\frac{v^2}{c^2}}, u'_z = 0$$

Similarly for Alien 2 relative to Alien 1

$$\begin{aligned} ct' &= \gamma\left(ct-\beta y\right), \\ x' &= x, \\ y' &= \gamma\left(y-c\beta t\right), \\ z' &= z. \end{aligned}$$

Then one can write

$$dt' = \gamma\left(dt-\frac{\beta}{c}dy\right), dx' = dx, dy' = \gamma\left(dy-c\beta dt\right), dz' = dz$$

the components of the velocity of Alien 1 relative to Alien 2

$$u'_x = \frac{dx'}{dt'}, u'_y = \frac{dy'}{dt'}, u'_z = \frac{dz'}{dt'}$$

becomes

$$\begin{aligned} u'_x &= \frac{dx}{\gamma\left(dt-\frac{\beta}{c}dy\right)}, u'_y = \frac{\gamma\left(dy-c\beta dt\right)}{\gamma\left(dt-\frac{\beta}{c}dy\right)}, u'_z = \frac{dz}{\gamma\left(dt-\frac{\beta}{c}dy\right)} \\ \Rightarrow\ & u'_x = \frac{\frac{dx}{dt}}{\gamma\left(1-\frac{\beta}{c}\frac{dx}{dt}\right)}, u'_y = \frac{\frac{dy}{dt}-c\beta}{1-\frac{\beta}{c}\frac{dy}{dt}}, u'_z = \frac{\frac{dz}{dt}}{1-\frac{\beta}{c}\frac{dz}{dt}} \end{aligned}$$

Noting that for Alien 2

$$\frac{dy}{dt} = \frac{dz}{dt} = 0, \frac{dx}{dt} = v$$

we find

$$u'_x = v\sqrt{1-\frac{v^2}{c^2}}, u'_y = -v, u'_z = 0 \Rightarrow u' = \sqrt{u'^2_x + u'^2_y + u'^2_z} = v\sqrt{2-\frac{v^2}{c^2}}$$

Prob 3 Problem # 1.4

Sol: We recall from HW 1.

$$x'' = -ct\sinh\left(\psi_v + \psi_{u'}\right) + x\cosh\left(\psi_v + \psi_{u'}\right),$$

so that for $x'' = 0$, one finds

$$x = ct\frac{\sinh\left(\psi_v + \psi_{u'}\right)}{\cosh\left(\psi_v + \psi_{u'}\right)} = ct\tanh\left(\psi_v + \psi_{u'}\right)$$

which leads to

$$u = \frac{dx}{dt} = c\tanh\left(\psi_v + \psi_{u'}\right).$$

Prob 4 Problem # 1.16

Sol: From what we derived in class

$$\frac{du}{dt} = \frac{du'}{dt'}\left(1-\frac{u^2}{c^2}\right)^{3/2}, \tag{1}$$

so that using Eq. (**??**), one finds

$$\frac{du}{dt} = \left(1 - \frac{u^2}{c^2}\right)^{3/2} f(\tau) \tag{2}$$

Noting that the proper time on the spaceship

$$d\tau = \sqrt{1 - \frac{u^2}{c^2}}dt, \tag{3}$$

we may rewrite Eq. (2) as

$$\frac{du}{d\tau} = \left(1 - \frac{u^2}{c^2}\right) f(\tau). \tag{4}$$

Introducing the transformation defined by

$$u(\tau) = c\tanh[\psi(\tau)] \tag{5}$$

we have

$$\begin{aligned} \frac{cd}{d\tau}\tanh[\psi(\tau)] &= \left(1 - \tanh^2[\psi(\tau)]\right) f(\tau) \\ \Rightarrow \left[\frac{c}{\cosh^2[\psi(\tau)]}\frac{d\psi(\tau)}{d\tau}\right] &= \frac{f(\tau)}{\cosh^2[\psi(\tau)]} \Rightarrow c\frac{d\psi(\tau)}{d\tau} = f(\tau) \\ \Rightarrow \psi(\tau) &= \frac{1}{c}\int_0^\tau f(\xi)\, d\xi. \end{aligned} \tag{6}$$

From Eqs. (3) and (5), we note that

$$\frac{dt}{d\tau} = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} = \cosh[\psi(\tau)], \frac{dx}{d\tau} = c\tanh[\psi(\tau)]. \tag{7}$$

For a constant acceleration (i.e. $f(\xi) = k$), so that

$$\psi(\tau) = \frac{1}{c}\int_0^\tau f(\xi)\, d\xi = \frac{k\tau}{c}, \tag{8}$$

we find for the time and position the first spaceship as function of the proper time

$$\frac{dt}{d\tau} = \cosh\left(\frac{k\tau}{c}\right) \Rightarrow t(\tau) = t_0 + \frac{c}{k}\sinh\left(\frac{k\tau}{c}\right), \tag{9}$$

$$\frac{dx}{d\tau} = \frac{dx}{d\tau}\frac{dt}{d\tau} = u(\tau)\frac{dt}{d\tau} = c\sinh\left[\frac{k\tau}{c}\right] \Rightarrow x(\tau) = x_0 + \frac{c}{k}\left[\cosh\left(\frac{k\tau}{c}\right) - 1\right]. \tag{10}$$

For $t_0 = x_0 = 0$, these equations can be rewritten as

$$\left(\frac{kt(\tau)}{c}\right) = \sinh\left(\frac{k\tau}{c}\right), \frac{kx(\tau)}{c} = \cosh\left(\frac{k\tau}{c}\right) - 1. \tag{11}$$

We recall the interval is given by

$$(c\Delta t)^2 - (\Delta x)^2 = (\Delta s)^2 \Rightarrow \left(\frac{c\Delta t}{\Delta s}\right)^2 - \left(\frac{\Delta x}{\Delta s}\right)^2 = 1. \tag{12}$$

which we may write in terms of the proper time, τ, as

$$\left(\frac{c\Delta t}{\Delta s}\right)^2 = 1 + \left(\frac{k\tau}{c}\right)^2 \Rightarrow \left(\frac{\Delta x}{\Delta s}\right)^2 = \left(\frac{k\tau}{c}\right)^2.$$

The parametric graph for the worldlines, for spaceship that is uniformly accelerating, $f(\tau) = k$, (the red curve)

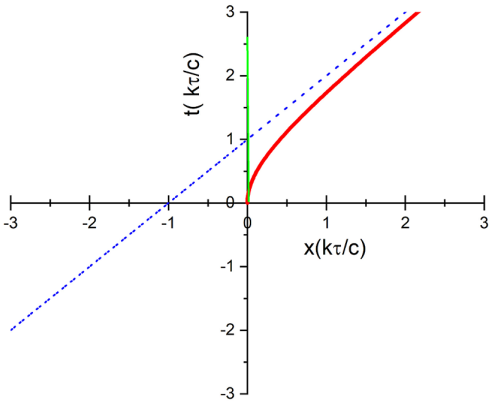

Figure 2: The worldline for the accelerating spaceship (red) and the observer on S (green). The dotted blue line is the horizon for the accelerating spaceship.

and for the observer on earth on S, $x = 0$, that is at rest on the S frame (green). The blue dotted line is the event horizon for spaceship. Events that are beyond the event horizon will never be seen by spaceship. For

$$x(\tau) = 10ly = 10c \times year, k = g = 9.8\frac{m}{s^2}, \frac{c}{g} = 3.06 \times 10^7 s$$

we have

$$\begin{aligned} \frac{kx(\tau)}{c} &= \cosh\left(\frac{k\tau}{c}\right) - 1 \Rightarrow \frac{k\tau}{c} = \cosh^{-1}\left(\frac{kx(\tau)}{c} + 1\right) \\ &\Rightarrow \tau = \frac{c}{g}\cosh^{-1}\left(\frac{gx(\tau)}{c} + 1\right) = 3.02 years \end{aligned}$$

We recall the Doppler shift

$$\frac{\nu}{\nu'} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}. \tag{13}$$

in terms of the wave length

$$\lambda' = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}\lambda. \tag{14}$$

For a light signal with wavelength, λ, send from the base we have $v = -u$

$$v = -u(\tau) = -c\tanh[\psi(\tau)] \tag{15}$$

so that

$$\lambda' = \sqrt{\frac{1 + \tanh[\psi(\tau)]}{1 - \tanh[\psi(\tau)]}}\lambda = \sqrt{\frac{1 + \tanh\left[\frac{k\tau}{c}\right]}{1 - \tanh\left[\frac{k\tau}{c}\right]}}\lambda. \tag{16}$$

This shows that as τ increases, $\lambda' > \lambda$ (redshift).

Prob 5 In Example 2.5, express the circumference (part b) and the area (part c) in terms of the distance D (part a) and determine D for the the maximum circumference and area. Find the maximum circumference and area of this sphere. Is your answer is consistent with what you know about the maximum circumference and surface area of a sphere with radius a.

Sol: