Unit I: Chapter 1 Vocabulary MATH 1010-K

1.1 Solving Problems by Inductive Reasoning

Conjecture: an educated guess based on repeated observations of a particular process or pattern. ***WARNING***: May or may NOT be true.

Inductive reasoning: drawing a general conclusion (a conjecture) from a specific example. Also called reasoning from the specific to the general; the small to the big; or from the inside out. {This is how we developed the Rules for Exponents, by taking specific examples and working out the general shortcuts they implied.}

Deductive reasoning: applying general principles to specific examples. Also called reasoning from the general to the specific; from the big to the small; or from the outside in. {This is how we used the Distributive Law to multiply and factor, patterns to identify types of equations.}

Natural Numbers: the set of numbers that begins with 1 and increases by ones. 1, 2, 3, . . .   Also known as (AKA): the Counting Numbers

Ellipsis: Three spaced periods (**. . .**) that means the sequence continues using the same pattern; or that some part of the set or sentence has been left out.

Premise: an assumption, law, rule, widely held idea, or observation.

Conclusion: a statement resulting from reasoning inductively or deductively from a premise.

Logical argument: consists of three parts: premise, reasoning, and conclusion.

1.2 Application of Inductive Reasoning: Number Patterns

Number sequence: a list of numbers having a first, second, third, and so on number in a repeating pattern (NOT random)

Term of a sequence: one of the numbers in the sequence.

Arithmetic sequence (AKA: Arithmetic Progression): Subsequent terms are found by adding the same value (+ or –) to the preceding term. {Grows by ADDITION} (We can generate the ordered pair of another point on a line by adding the rise to the y-coordinate and the run to the x-coordinate of a known point on a line, such as when we use the slope intercept form of a linear equation to graph the line. Arithmetic sequences are linear)

Common difference: the value added to a term in an arithmetic sequence to produce the subsequent term.

Geometric Sequence (aka geometric progression): subsequent terms are found by multiplying the same value (+ or –) times the preceding term. {Grows by MULTIPLICATION.} (This type of growth produces a NON–LINEAR graph, that is, a curved shape, such as a parabola.)

Common Ratio: the factor multiplied times a term in a geometric sequence to produce the subsequent (next) term.

1.3 Strategies for Problem Solving

Polya's Method

George Polya (1888–1985) developed a four-step, methodical approach to solving problems and set forth in his book *How To Solve It* (1973).

1. Understand the problem: understand what you are being asked to find. You must read and analyze carefully, perhaps several times, until you can confidently answer the question, "What must I find." This is what Dr. O is talking about when he says you need to read with understanding and be able to state your goal.

2. Devise a plan: You can attack a problem in several ways. Decide what plan is appropriate for the particular problem you are trying to solve. In mathematical-type word problems, this step produces an equation or series of equations that will lead to your goal.

3. Carry out the plan: Once you have determined your approach, carry out your plan, you may run into roadblocks and dead ends, but be persistent. Once you find a numerical answer, don't forget to check it in your original equation.

4. Look back and check: Check your result to see if it is reasonable. Does it satisfy the conditions of the problem? Have you answered all of the questions the problem asks? Con you solve the problem in a different way and come up with the same answer? This is where you apply Dr. O's Three R's of problem solving.

Polya, G. (1973). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton Paperback, Princeton University Press.

What are some tools or strategies that can be useful to solve problems (math or real problems)?

1.4 Numeracy in Today's World

Numeracy: The mathematics version of literacy for reading. Numeracy is the ability to understand and make meaningful use of fundamental number concepts and tools, such as a calculator or spreadsheet.

Graphing calculator: A tool that will display mathematical information on a graph as well as perform calculations involving addition/subtraction, multiplication/division, exponents/roots, and grouping. The required calculator for MATH 1010 is one of the TI–83 or TI–84 calculator variants (83+, 84+, Silver, etc.)

Estimation: A concepts that allows you to quickly determine an approximate value or results when an exact answer is not required.

Calculation: requires following the Order of Operations so that the correct result will be obtained by numerate persons. The Order of Operations is the process by which a calculation involving more than one operation and involving grouping.

Graphs: a visual representation of data and may be a circle graph (sometimes called a pie-chart), a bar graph, or a line graph.

Tables: Data listed in rows and columns; always read row first, then column.

Unit I: Chapter 2 Vocabulary MATH 1010-K

2.1 Symbols and Terminology

Set: a collection of objects. A set is designated by a capital letter: set A = {1, 2, 3, a, b, c}

Element: Any one object in a set. Both 2 and c are elements of A.

Three Methods of designating sets:

1. Verbally describes the set in words,

2. Numerically lists all the elements of the set, and 3. Symbolically uses Set Builder Notation

Null Set (AKA the Empty Set): a set with no elements. Designated by **{}** or **ø** Symbols

$\in$ is an element of a given set. For set A above, $3 \in A$

$\notin$ is not an element of a given set. For set A above, $d \notin A$

Examples: Let E be the set of all letters in the English alphabet. Then, $m \in E$: "m is an element of the English alphabet."

$\pi \notin E$: "pi is not an element of the English alphabet."

Cardinal Number/Cardinality: the number of elements in a set. Pattern:

n(A) read "The n of A," where n is a Natural Number and A is the name of a particular set. n(A) means the number of elements in set A. For set A = {1, 2, 3, a, b, c}, n(a) = 6

Example: If K = {2, 4, 8, 16}, then the n(K) = 4.

What is the n(E) for E = English alphabet?

Finite set: When the n(A) is a Whole Number (0, 1, 2, . . .). A set is finite when the number of elements can be counted.

Infinite set: When the number of elements cannot be count, for example, R, the set of Real Numbers is infinite because its elements cannot be counted.

Equal sets: Two sets are equal if the have exactly the same elements.

2.2 Venn Diagrams and Subsets

Universe of Discourse: all things under discussion at a given time. Usually called *the Universal Set*, designated U, and might change from one problem to another.

Venn Diagrams: developed by John Venn (1834-1923). U is a rectangle (a square is a special type of rectangle). Other sets within U are circular regions (sometimes ovals or other shapes).

Complement of a set: for a set A within U, all of U outside of A is the Complement of A, labeled A' and read "A prime" (think "the complement of A"). The complement of a set COMPLETES the Universal set. Set A union A' = U.

Subset: When all of the elements of a set are also elements of another set. If all the elements of A are also elements of B, then A is a subset of B, symbolically $A \subseteq B$. Since every set is a subset of itself, for set A to be a subset of set B, then n(A) < n(B). The Null Set is also a subset of every set.

> **N.B.** the subset symbols looks somewhat like the $\leq$ inequality symbol; so think the number of elements in a subset are less than or equal to the number of elements in the main set.

Proper subset: If $A \subseteq B$ and $A \neq B$, then $A \subset B$

For set A to be a proper subset of set B, then A must first be a subset of B AND have fewer elements than set B. n(A) < n(B).

> **N.B.** the subset symbols looks somewhat like the < inequality symbol; so think the number of elements in a subset are less than the number of elements in the main set.

The Null Set is a proper subset of EVERY set except itself. No set has fewer elements than the Null Set; therefore, the Null Set has no proper subsets.

Every set except ø has at least 2 subsets, ø and itself. (regular subsets, not proper subsets; the number of proper subsets is one less than the number of regular subsets)

Examples: If set A = {1}, then A has 2 subsets, the Null and itself.

Let set B = {1, 2}. How many subsets does B have? Four: ø, itself, {1}. and {2}.

Let C = {a, b, c}. Its subsets are: ø, itself, {a}, {b}, {c}, {a, b}, {a, c}, and {b, c}, a total of 8. To examine the data and try to find a pattern, we will use a table.

Number of elements	1	2	3	4
Number of subsets.	2	4	8	?

If the rate of change stays the same, how many subsets do you expect for a set with 4 elements? Why?

As the number of elements in the set increases by +1, the number of subsets doubles (previous number of subsets * 2). Count the number of factors of 2 for each group of subsets: 2 subsets have 1 factor of two; 4 subsets have 2 factors of 2; 8 subsets have 3 factors of 2. Did you notice that the number of factors of 2 is the same as the number of elements in the set? How would you explain this pattern so that you could calculate the number of subsets when you know the number of elements in a set?

2.3 Set Operations (and Cartesian Products)

Intersection of sets: the set formed by taking the common elements from two sets. For an element to be in the intersection, it must be in BOTH of the original sets. Symbol is $\cap$ and the *keyword* is "**and**."

Example: Let set A = {1, 2, 3, 4} and set B = {3, 4, 5}. Since 4 is an element of set A **and** set B, then 4 is an element of $A \cap B$ (the intersection of A with B)." What other element is in $A \cap B$? What is the $n(A \cap B)$?

Union of sets: the set formed by taking all of the elements from one of two sets, then adding all the elements of the second set that are not already listed. For an element to be in the union, it is sufficient for it to be in either one of the original sets. Symbol is ∪ and the key word is "**or**." If an element is in one of the two sets, it is in the Union

Example: Let set A = {1, 2, 3, 4} and set B = {3, 4, 5}. Even though 5 is an element of only set B, it is still an element of A ∪ B (the union of A and B)."

What is the complete set for A ∪ B (the union of A with B)?

A ∪ B = {

n(A ∪ B) =

RULE of thumb: an Intersection generally produces a set smaller than either original while a Union produces a set larger than either original set.

Difference of sets: the set obtained by removing all common elements of the second set from the first set.

Example: Let set A = {1, 2, 3, 4} and set B = {3, 4, 5}. Then, A – B = {1, 2}. What is n(A – B)? Does B – A produce the same difference of sets?

Cartesian Product of sets: a process that pairs each element of set A with all the elements in set B to produce ordered pairs (Similar to the Distributive Law but do NOT multiply the numbers).

Example: Let set A = {1, 3} and set B = {2, 4, 6} then A×B (read "A cross B") produces the following ordered pairs: A×B = {(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)}.

De Morgan's Laws: Two statements investigated by August De Morgan, a

British logician who lived from 1806 until 1871, and found to be true. For any sets A and B,

1. The complement of the intersection of two sets is equal to the union of the complements of the two sets.

   Symbolically, (A ∩ B)' = A' ∪ B'

2. The complement of the union of two sets is equal to the intersection of the complements of the two sets. Symbolically, (A ∪ B)' = A' ∩ B'

2.4 Surveys and Cardinal Numbers

Cardinal Number Formula: For any two sets A and B, the following is true:

n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

Example: Let set A = {1, 2, 3, 4} and set B = {3, 4, 5}. n(A) = 4 and n(B) = 3.

(A ∪ B) = {1, 2, 3, 4, 5} and n(A ∪ B) = 5

(A ∩ B) = {3, 4} and n(A ∩ B) = 2

Substituting the numbers into the formula n(A ∪ B) = n(A) + n(B) – n(A ∩ B) gives us: 5 = 4 + 3 – 2 True of false?

Symbols used in this document are from: http://rapidtables.com/math/symbols/Basic_Math_Symbols.htm