Chapter 3: Introduction to Logic

Logic: any formal system that abstracts the form of statements away from their content in order to studying their truth value.

Symbolic Logic: A system that assigns letters to represent statements and symbols to represent words such as *and*, *or*, *not*. Only used for statements that involve facts, not opinion. One of the early inventors is Gottfried Leibniz, 1646 – 1716.

Statement: A declarative sentence (of fact) that is either true or false, but not both simultaneously.
Compound statement: A statement formed by joining 2 or more statements into one. **Component Statement**: one of the statements that make up a compound statement.

Logical Connectives: Sometimes referred to simply as connectives, words used to form compound statements. **Examples**: *and, or, not, if . . . then*

Negations: a statement is a negation of a statement if and only if it has the opposite truthe value. A negation MUST have the opposite truthe value from the original statement: ~~the negat~~ion of a true statement is false, and the negation of a false statement is true.

Conditional Statement: a compound statement using the "if . . . then" connective.

Symbolically: $p \Rightarrow q$ Read: "If p, then q" or "p implies q"

In $p \Rightarrow q$, p is the antecedent statement and the q is the consequent statement. `Does NOT imply a cause and effect relationship. A conditional is F (false) ONLY when the antecedent (p) is T (true) and the consequent (q) is F. A conditional is automatically T when p is F and, also, when q is T.

Quantifiers: indicate how many of a particular situation exists. Two types:
.....**Universal Quantifiers**: all, each, every, no, none; and
.....**Existential Quantifiers**: some, there exists, (for) at least one

Types of Compound Statements

Name	Symbol	example	How read	Key Idea
Conjunction	$\wedge$	$p \wedge q$	"p and q"	BOTH
Disjunction	$\vee$	$p \vee q$	"p or q or both"	inclusive or
Negation	~	~p	"not p"	opposite

Truth Tables: a symbolic method of determining the truth value of compound statements. When the results column of a truth table is all T, then the compound statement is a tautology.

Equivalent Statements: have the same truth value in every possible situation.
Symbol: ≡ Means : identical to

Numeration Systems: the various ways of symbolizing and working with the counting numbers.

Numerals: the symbols used to represent the numbers.

Hindu-Arabic system: the modern decimal (base 10) number system.

H–A numerals: {1, 2, 3, 4, 5, 6, 7, 8, 9, 0} aka: digits.

Roman Numerals: the system used in the Ancient Roman Republic and Empire, also base 10. Roman numerals are still used in a primarily decorative fashion. See p. 141 for the Roman numeral symbols and the Special Features of the Roman System.

I expect you to be able to correctly write and interpret numbers in Roman numerals, such as a year or the number of a Super Bowl.

SYMBOL	MEANING	EXAMPLE
{ }	is a set	$S = \{4,5\}$
$\in$	is an element of	$s \in S$
$\notin$	is not an element of	$s \notin T$
$\subseteq$	is a subset of	$S \subseteq T$
$\subset$	is a proper subset of	$S \subset T$
$\cup$	union	$S \cup T$
$\cap$	intersection	$S \cap T$
$\varnothing$	the empty set	$\{2,3,4\} \cap \{5,6,7\} = \varnothing$

Recap: Conditional Statements

Conditional $(p \to q)$	If I am sleeping, then I am breathing
Converse $(q \to p)$	If I am **breathing**, then I am **sleeping**.
Inverse $(\sim p \to \sim q)$	If I am not sleeping, then I am not breathing.
Contrapositive $(\sim q \to \sim p)$	If I am **not** breathing, then I am **not** sleeping.