Operations with Real Numbers

Arithmetic officially has 4 operations: addition, subtraction, multiplication, and division.
Algebra officially recognizes only two: addition and multiplication.
Why? Arithmetic is primarily concerned with positive numbers while algebra uses the set of Real Numbers. A **Real number** has 2 characteristics: a *sign* (the number's direction from 0 on the number line) and a *value* (the number's distance from 0 on the number line).

Since algebra uses signed numbers, what does this mean for operations?
1. Algebra defines subtraction as a combining 2 numbers by adding the opposite of the second term; therefore, subtraction is defined as a form of addition.
Ex: $4 - 7$ means $4 + (-7)$
This definition allows us to write any subtraction item as an addition item and apply the rules for adding numbers.

2. Algebra defines division as multiplying by the reciprocal of the second factor; therefore, division is a form of multiplication.

Ex. $8 \div 3$ means $8 \bullet \frac{1}{3}$
This definition allows us to write any division item as a multiplication item and apply the rules for multiplying numbers.

So, what are these addition and multiplication rules for signed numbers?
Thought you'd never ask:

Adding two numbers with:
Same signs (both positive or both negative): Add the values and keep the sign.
Examples: $4 + 7 = 11$ and $-4 + (-7) = -11$

Opposite signs (one positive and the other negative): Take the difference of the two values and keep the sign of the one with the larger (absolute) value.
Examples: $4 + (-7) = -3$ and $-4 + 7 = 3$

To subtract two signed numbers, apply the definition of subtraction and then use the addition rules.
Definition of Subtraction: $a - b \Rightarrow a + (-b)$ Read "a minus b means a plus the opposite of b"
Examples: $4 - 7 \Rightarrow 4 + (-7) = -3$
$-4 - (-7) \Rightarrow -4 + 7 = 3$
$-4 - 7 \Rightarrow -4 + (-7) = -11$
$4 - (-7) \Rightarrow 4 + 7 = 11$

Multiplying two numbers with:
Same signs (both positive or both negative): multiply the values to find the product, and the product is ALWAYS positive.
Examples: 8 * 3 = 24 and −8 * (−3) = 24

Opposite signs (one positive and the other negative): multiply the values to find the product, and the product is ALWAYS negative.
Examples: −8 * 3 = −24 and 8 * (−3) = −24

To divide two numbers, convert from division to multiplication and use the same multiplication rules. When the two numbers have the
Same signs (both positive or both negative): divide the values to find the quotient, and the quotient is ALWAYS positive.

Examples: $8 \div 3 \Rightarrow 8 \bullet \frac{1}{3} = \frac{8}{3}$ and $-8 \div (-3) \Rightarrow -8 \bullet \left(\frac{-1}{3}\right) = \frac{8}{3}$

Opposite signs (one positive and the other negative): multiply the values to find the quotient, and the quotient is ALWAYS negative.

Examples: $8 \div (-3) \Rightarrow 8 \bullet \left(\frac{-1}{3}\right) = \frac{-8}{3}$ and $-8 \div 3 \Rightarrow -8 \bullet \left(\frac{1}{3}\right) = \frac{-8}{3}$

Summary:
Adding two numbers
a) with the same signs, add the values and keep the sign;
b) with opposite signs, subtract the smaller value from the larger value and use the sign of the larger value.
Subtracting signed numbers, convert to addition and use the addition rules.

Multiplying or dividing two signed numbers
a) if the two numbers have the same sign, the result is <u>always</u> positive;
b) if the two numbers have opposite signs, the result is <u>always</u> negative.