

# Inventing Displays

## Unit

# 1

## Mathematical Concepts

- Measurement is a repeatable process.
- Any measure of an attribute is composed of signal and noise. Signal comes from the true measure of the attribute; noise comes from measurement error.
- The shape of displayed data arises from choices made by the designer of the display. Different choices result in different shapes for the same data.
- Every display highlights some features of the data and backgrounds other features.

## Unit Overview

Unit 1 focuses on the development of representational and meta-representational competencies, meaning that students progress from case-based interpretations of data representations to those involving characteristics of the aggregate. Students learn that the shape of the data arises from the choices that designers make to show and hide aspects of the data.

### Day 1: Taking Measurements

Students begin by generating the data for this unit. For example, students might measure your arm span or the perimeter of a table. Everyone measures the same object, but they measure independently (meaning measurements are secret until the dataset is complete) and with a crude tool (for example, a short ruler). Variability in measurements lays the groundwork for subsequent discussion about measurement and the design of data displays. If time permits, students re-measure the same object with a better tool (for example, a meter stick).

### Day 2: Inventing Displays

Next, students work in pairs to notice trends in their class measurements and invent a display that highlights those trends. Variability, this time in the design of student displays, is once again a good thing—fodder for discussions about design choices and how they affect what a display shows and hides.

### Days 3 and 4: Comparing Displays

Students compare and contrast a small number of carefully chosen student displays. What does each display help readers/viewers understand? What does it hide? Students learn that every representation/display highlights

## Contents

Mathematical Concepts	1
Unit Overview	1
Materials & Preparation	3
Mathematical Background	4
Instruction	
Taking Measurements	7
Inventing Displays	11
Comparing Displays	15
Formative Assessment	22
Unit Quiz	25
Unit Quiz Scoring Guide	29

## Unit Overview

## Unit 1

some features of the data and backgrounds other features. Students also relate qualities of display to the processes of measurement, especially when they consider what might happen if they measured again or if another class measured the same attribute (e.g., another class measures the same teacher's arm span or the perimeter of the same desk).

### **Days 5 and 6: Formative Assessment**

Students respond to a brief quiz. After looking at student responses and a scoring guide, several items are selected as the basis for further instruction via a whole-class conversation. Student responses that represent different ways of thinking, according to the Data Display and Meta-Representational Competence construct maps, are deliberately compared and contrasted.

## Read

### □ Unit 1

Start by reading and working through the unit materials to learn the content and become familiar with the activities.

*Note:* The activities in Unit 1 rest on a firm grasp of principles of measurement, such as: a measure is obtained by iterating (repeatedly translating) a unit and a measure is a quantity expressed by accumulating units. To make some of these key ideas visible to students, provide opportunities to consider these and related aspects of measure, perhaps by measuring a distance using whatever tools they invent. See Linear Measure units on [datamodeling.org](http://datamodeling.org) for additional information.

### □ Sample student thinking and classroom talk

Reread Inventing Displays (pp 10-12), Comparing Displays (pp 13-16), and Relating Display to Process (pp 17-18) to anticipate the kinds of ideas and discussions you will likely see during instruction.

### □ Data Display and Meta-Representational Competence construct maps

Read the maps and/or go to [Modelingdata.org](http://Modelingdata.org). Note the milestones of progress in student thinking about the mathematics of representational and meta-representational competencies.

### □ Read Discussion Guide and Display Comparison Planner on

[Modelingdata.org](http://Modelingdata.org) to plan for guiding a whole class conversation where students compare what different displays show and hide about the data, and that helps students understand the mathematical ideas used to create different shapes for the same data.

## Gather

### For the class

#### □ 1 short ruler, 15 cm. or 6 in. long

Why a short ruler? A short ruler will be moved more often to measure the person's arm span or head circumference, so the measurements will tend to be less precise and, therefore, more spread out.

#### □ 1 meter or yardstick (optional)

Students can later measure again with one of these tools. The measurements will be less variable, because the tool does not have to be moved as often.

## Materials & Preparation

## Inventing Displays Unit 1

### **For every group of two students**

- Sticky notes  
Students independently record measurements on sticky notes (one sticky per measurement, one measurement per person, different colors if possible). Alternatively, index cards or scraps of paper can be used.
- Chart paper  
One large sheet per pair of students for inventing displays.

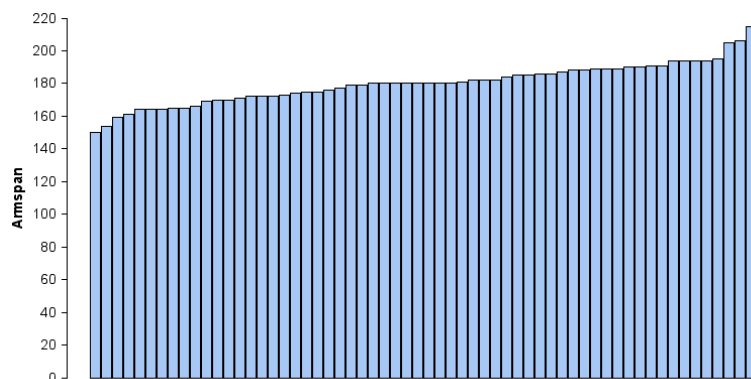
## Mathematical Background

## Inventing Displays Unit 1

The shape of the data evident in a display reflects the choices made by the designers about which characteristics of the data to emphasize. Numerical data consists either of categorical counts of things (e.g., the number of students in a classroom) or of measurements of an attribute of those things (e.g., the height of each student in a classroom). The data obtained in this unit are measures, so they can be scaled. A scale orders the magnitudes of measured units by constant intervals, such as 1 cm., 2 cm., and so on.

### Displays based on order

Displays can be constructed to showcase the order of the values obtained. Ordered values can be represented as *lists* or as *case-value plots* (see below). The case-value plot represents each value by a length proportional to the magnitude of the value. It orders the values from least to greatest. Plateaus on the graph represent similar values.



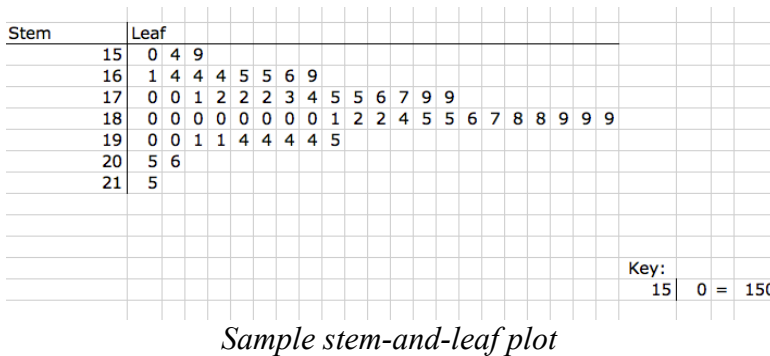
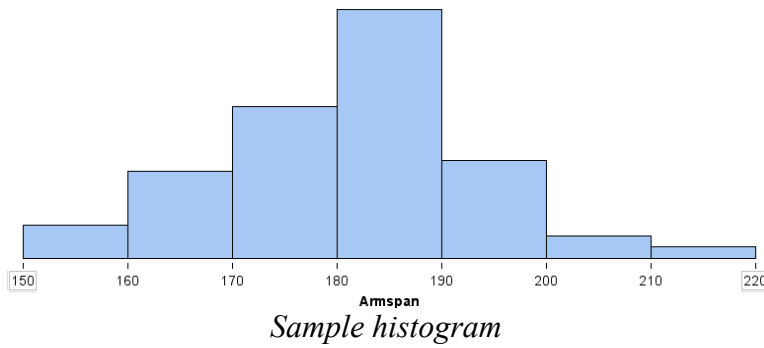
ordered by Armspan  
*Sample case-value plot*

### Displays based on class (interval) and count (frequency)

Often we wish to treat distinct data values as alike—as belonging to the same class of values, such as “the 150’s.” These displays show the frequency of each class, either explicitly by counts or implicitly by relative heights of the plotted values. A histogram is perhaps the most familiar form of this type of plot, although other plots can be produced. The classes ideally are well defined, mutually exclusive and of common interval. See the next page for examples: histogram, class (interval) frequency and stem-and-leaf plots.

# Mathematical Background

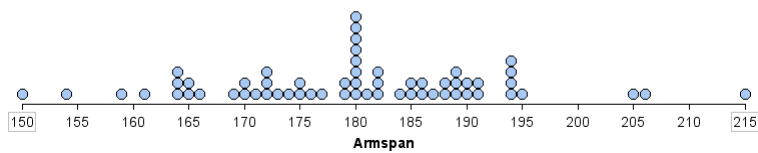
# Inventing Displays Unit 1



Note that rotating the stem-and-leaf plot produces a plot much like the frequency plot.

### Displays based on scale and count

Displays based on scale align each value along a *common scale*, allowing viewers of the display to observe gaps or discontinuities in the data. These displays reflect the frequency of each value, either explicitly by count or implicitly by relative heights of the plotted values. Notice that in the dot plot below gaps in the data are evident. These gaps were hidden by the classes (groups of 10) used in the previous displays (e.g., the histogram).



Sample dot plot

## Instruction

## Inventing Displays Unit 1

### Taking Measurements

In this task, students explore the important concept of repeated measure. Students use a crude measuring tool to measure a particular attribute of a particular person, for example, the span of your arms or the circumference of one student's head. Or, if you prefer a more impersonal measure, the perimeter of a table might be taken. The discussion in this task provides an opportunity for students to consider the meaning of repeated measure. Why is it helpful to measure the same thing multiple times? Why might different people get different measurements?

### Whole Group

1. Give a brief overview of curriculum goals, structure, and norms.
2. Introduce the task: repeated measure of teacher arm span.

For example:

*The goal is for each person to measure the same thing (my arm span) using the same tool (this six-inch or 15 cm. ruler). Record your measurement on a sticky note: one measurement per person and per note. Don't look at other people's measurements until everyone is done.*

Students will need to take turns using the ruler and measuring your arm span, so it may be helpful to have another task on hand for those students waiting to measure.

If time allows, students can generate a second dataset for use in Units 2, 3. For example, students might use a meter or yardstick to re-measure the same attribute (e.g., your arm span). The rationale is that better tools reduce variability but leave the center relatively unchanged.

*Note:* It is important that:

- Every student measures the same attribute of the same person or of the same object.
- Students use a crude measuring tool, such as a short ruler, before they use a better tool, such as meter stick or tape measure.

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment



## Instruction

## Inventing Displays Unit 1

- Students are allowed to make mistakes. In this activity, variation in measurements is a good thing and lays the groundwork for important conversation about sources of variability and trends in measurements.
- Student measurements are independent, meaning students do not know the value of the measurement found by classmates until all data have been collected.
- Students generate a large dataset (30 measurements or more). For this reason teachers sometimes involve more than one class in this activity. Or use the class measurements and then add some plausible values so the overall distribution is roughly symmetric. (See sample datasets for Unit 1 at [modelingdata.org](http://modelingdata.org))

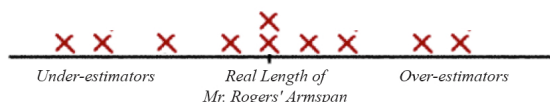
Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

### 3. Support student thinking about the meaning of the measurements they are about to make.

- Before students measure, support students to think about how their measurements might be related to the actual length of the attribute. Use questions such as these:
  - Q: Why don't we just have one person do the measurement and tell us what they got?
  - Q: How could we use this tool to measure this body part?
  - Q: What should we keep in mind when we measure? Why is it important to keep this in mind?
  - Q: What do you think will happen when we measure? What do you think we will get? Why?
  - Q: Compared to the real length of her arm span, where will our measurements be (less than the real length, more than the real length, or the real length)? How much less/more do you think our measurements will be than the real length (a lot less/more, a little less/more)?
- If necessary, to generate further classroom conversation about the meaning of the measurements, use an empty number line:
  - Draw a line on the board and label a point as, "the real length of (name-of-person)'s arm span."
  - Then ask students to come up and use an X to represent a possible value of the measurement.

## Instruction

## Inventing Displays Unit 1



- iii. Ask the student to tell why she or he chose to make a mark there. It might be useful to use language for marks to the right of the arm span as “over-estimators” and to the left of the arm span as “under-estimators.” Some students may believe that all the possible values will be exactly on the real-length mark.

**Taking Measurements**  
 Inventing Displays  
 Comparing Displays  
 Formative Assessment

## Individual Work

4. **Allow students to complete their measurements.**
5. **Listen and watch for differences in measurement and method to highlight in the whole-group discussion.**

*Note:* If students do not agree about method of measure or units of measure, the resulting distribution of measurements will reflect a blend of different methods of measure, individual differences in interpreting the task, and other chaos. This is fine, because it leaves room for wondering why the shape of the data changes when the measurements are conducted more uniformly and/or with greater accuracy.

## Whole Group

6. **Share students’ measurements; describe and discuss their variability.**

Help students notice and begin thinking about sources of variability and trends in their measurements. For example, publicly display all sticky notes without organizing them (the students should generate the organization, not the teacher!) and ask questions such as these:

- Q: What were you trying to find out by measuring?  
 Q: What do you notice about the measurements? Would you say that we all got the same value? How different are our values?  
 Q: Why did we get different measurements?  
 Q: Do you think your measurement is close to the real length of (name-of-person)’s arm span? Why?

## Instruction

## Inventing Displays Unit 1

*Note:* If time permits, it can be helpful to record all the measurements in a table and print copies for each pair of students to use in the next activity. Just be sure the data are presented as a collection, not ordered or otherwise structured. Here is a place where you can begin to create a public record of student thinking by recording some of the student responses to each of the questions.

- 7. Have each student repeat his or her measurement with a tool that will produce less variability, such as a meter stick or tape measure.**

Put each measurement on a different colored sticky note and record or store for later use. Use the first set of measurements for the next activity, Inventing Displays.

*Note:* Unit 4, Day 1, also outlines this activity. But it is often helpful to have a second set of data handy when students invent statistics in Units 2, 3. There is usually less error in measurement when students use a meter stick or tape measure, because they do not have to iterate as much. But the interpretation of the process is the same: Signal comes from the real length of the object being measured and variability from errors in the process of measurement.

### Taking Measurements

Inventing Displays

Comparing Displays

Formative Assessment

**Inventing Displays**

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

In this task, students work in pairs to identify trends in the class's measurements and invent displays that highlight those trends. They consider patterns in their data, as well as the form and function of their displays. Your goal is to support invention and variation in student displays and to encourage students to make important decisions on their own, such as what trend or relationship they are trying to show and how best to design their display in order to highlight that trend or relationship.

**Whole Group****1. Introduce the task.**

- a. Post the data in a central location or provide students with a copy of the (unordered, randomly arranged) measurements. Distribute chart paper to each pair of students.
- b. Tell students to make a display on the chart paper that shows other people all of the measurements at a glance. Explain that the display should help other people quickly see anything they think is important or worth noticing, such as a trend (a tendency, a pattern) or relationship among the measurements. Tell students to write large enough so everyone can see.

If students ask for additional direction (e.g., Should I make a graph?), let them know that the choice of kind of display is up to them. Leave the task appropriately ambiguous to allow for a variety of designs.

*Note:* If students ask to use computer tools to create their displays, suggest that, in this circumstance, paper and pencil leads to more invention. We suggest this because paper and pencil often make the important issue of interval more visible. For example, students may create “bins” for values, intervals that affect the “shape” of the data. Students may also juxtapose the bins without regard to the entire range of the interval. Students might arrange values in order, such as 10s, 20s, and then juxtapose 40s if there are no values in the 30s bin. The resulting display highlights clumps of values but makes “holes” in the data invisible.

## Instruction

## Inventing Displays Unit 1

*Note:* Students tend to design a variety of unconventional displays (see the Students' Ways of Thinking box for examples). This is an intentional and productive feature of the unit. Subsequent discussion about what different representations show and hide helps students develop an appreciation of the “shape” of the data.

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

## Pairs

**2. As students work on their displays, interact with them and look for displays to highlight in tomorrow’s whole-group discussion.**

- a. Observe and talk with students. As needed, use questions to help students identify and make important decisions about the purpose and design of their displays:

Q: What do you think is interesting in the data set?

Q: What trend, shape, or strange values do you notice?

Q: What is the first thing you want someone else to notice in your display? How have you shown that?

- b. Look for displays to highlight in tomorrow’s whole-group discussion. See the box below for common examples of student thinking.

**Construct: DaD1-DaD4**

This task engages students in thinking at the early levels of the Data Display construct.

### Students' Ways of Thinking: Inventing displays

**Noticing Order (Lists).** Some students tend to focus on the order of the data (for example, the fact that some values are greater than others).

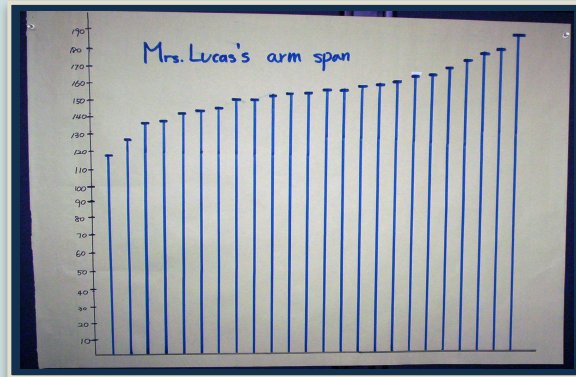
Typically, they arrange data into lists.



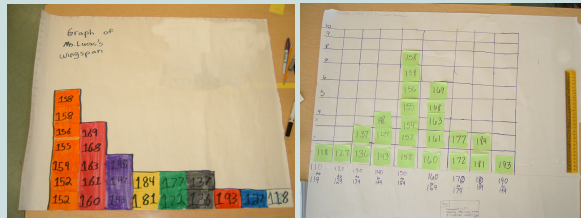
# Instruction

# Inventing Displays Unit 1

**Noticing Order (Case-value Graphs).** Some students use lengths of lines to show order. These displays are called case-value or array graphs. They use relative length to convey at a glance differences in the magnitudes of the measurements.



**Noticing Similar Values.** Some students notice that values tend to cluster. Their displays reflect this organization. Two examples are provided below. Notice that the first orders the groups or “bins” of data from most frequent to least frequent. The second is closer to convention in that it first orders the measured values and then shows their count (frequency) to allow viewers to see a center clump in the data. These are examples of grouped-values displays.

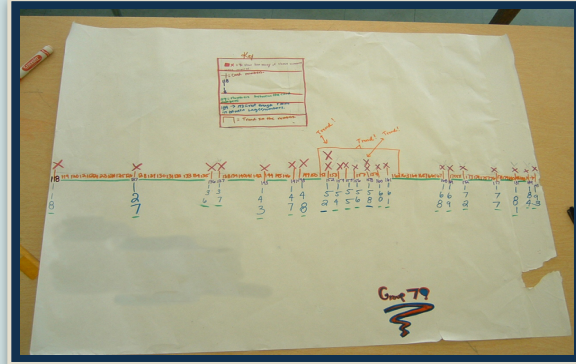


Taking Measurements  
 Inventing Displays  
 Comparing Displays  
 Formative Assessment

# Instruction

# Inventing Displays Unit 1

**Noticing Scale.** Other students notice order and frequency of data values, but they also represent distances between individual values on a scale. Hence, they can see holes and clumps in the data.



Taking Measurements  
 Inventing Displays  
 Comparing Displays  
 Formative Assessment

**Comparing Displays**

In this task, students notice and compare important features of their displays, such as how they used order, count, group (interval) and scale to represent the data. Students interpret what each display allows everyone to “see” and what each display “hides.” The aim is to help students recognize that different senses of the data are tied to how the data are displayed, and that representational choices entail trade-offs. Your goal in this discussion is to support student observations and comparisons between displays by selecting and juxtaposing mathematically productive examples (based on the construct), highlighting their mathematical components, and using questions to generate and guide discussion. The conversation concludes with student consideration of how the shape of data relates to processes of measurement.

**Whole Group****1. Describe and compare displays.**

- a. Publicly exhibit student displays and have other students either write or report verbally about one aspect of the measurements that the display helps make more visible—what it shows-- as well as one aspect of the measurements not evident from just looking at the display—what it tends to hide.

You might ask students to give their display to another pair, and then have each student pair describe the qualities of the display they notice. Many teachers find this pair format preferable.

*Note:* You may feel tempted to validate student displays as simply right or wrong without discussion of key elements, or to adopt a turn-taking strategy that allows every pair of students to share their displays. Avoid these temptations! Your aim in this discussion is to help students learn how attention to order, count, group, and scale alter the shape of the data. And, help students learn that every representation highlights some features of the data (shows) and backgrounds (hides) others.

Many teachers struggle to have all students share and still have a productive discussion. Some strategies that others have found helpful are:

Taking Measurements  
Inventing Displays  
**Comparing Displays**  
Formative Assessment



## Instruction

## Inventing Displays Unit 1

- Display all displays (so they remain up for the discussion)
- Choose displays that are examples of the kind of strategies students used. For example, some displays may focus on order, others on groups of values and yet others on scale.
- After each presentation, ask “did anyone else do something similar?” and have students explain how theirs are similar, OR order them yourself before the presentation. Students with similar displays present, then discuss all of those at once.

- b. Use questions to generate discussion and comparison of displays.

For example, use *evaluation and comparison questions* to help students consider how decisions in making displays affect what one sees (or does not see) about the data:

- Q: What does the display do a good job of helping other people notice? What does it make less noticeable?  
 Q: How did the designer show/hide feature X?  
 Q: What can you see with this display that you can't see as easily with this other display?

Use *what if questions* to help students see how design decisions affect what they perceive about data:

- Q: What would display X look like if the whole class got 193?  
 Q: What would display X look like if the maximum measure were 220 cm? How about display Y?

Use *translation questions* (questions that involve tracing cases or groups of values from one display to another) to help students understand how some qualities of the data can be enhanced in one view and hard to see in another, and that we generally need more than one display to get a good grasp on a batch of data:

- Q: If we want to find this group in this display (display X), what do I look for here (display Y)? Can I find them easily?  
 Q: I can see in display X that several people got 152 cm. for the armspan. How would I see that on display Y? Could I see it easily?  
 Q: No one got a measure of 160 cm. How can I see that on display X? Can I see it on display Y? How?

Taking Measurements  
 Inventing Displays  
 Comparing Displays  
 Formative Assessment

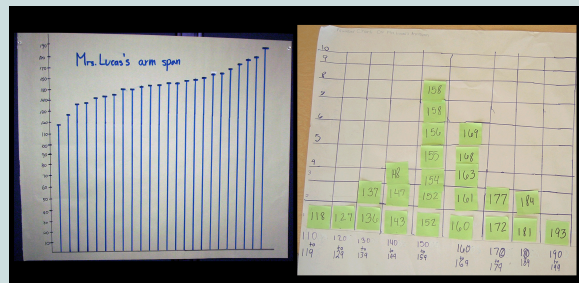
**Construct: MRC2, MRC3, & MRC4**  
 These levels include examples of student thinking and, in the videos, teacher responses to those forms of thinking.

## Instruction

## Inventing Displays Unit 1

## Students' Ways of Thinking: Comparing different displays

In this example, a teacher asked students to compare the case-value display and the grouped-values display. She wanted students to see how their choices made the same data look different.



A student said, “I think theirs (the case-value graph) are started from the bottom and goes up but doesn’t come back down, because they didn’t do it like ours. They just did bars instead of doing different columns like what we were doing, 90s, 80s, 70s, 60s, and so on.” Another student explained how design choices made differences in shapes of the two graphs. She mentioned that the case-value graph went up like stairs because its inventors ordered measurements from least to greatest and used lines to represent magnitude of each measurement. However, the grouped-values graph made a “mountain shape” because its creators used bins of 10s.

The teacher followed up with a ‘what if’ question: “What would the case-value graph look like if the whole class got 193?” A student answered, “It would just be the same line all across.” After some more conversation, the teacher posed a translation question: “If I want to find this group in there (refers to grouped-values graph), what do I look for here (refers to the case-value graph)? Can I find them easily?” A student said that it was very difficult because they had to go across to read each value from the y-axis. Another student said that she would look for plateaus to find similar values on the case-value graph.

## 2. Direct student attention to important ideas, such as clumps and holes, when comparing displays.

### Classroom Talk: Directing student attention to important ideas

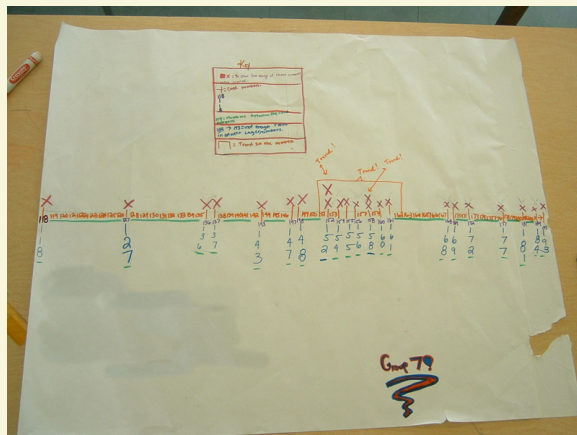
Sometimes a teacher has to be very direct and make sure students notice a worthy comparison. In this example, a group of students was interested in values that were missing, so they drew a number line and differentiated observed values and missing values by using colors and text sizes. Also,

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

## Instruction

## Inventing Displays Unit 1

the student-inventors put Xs to represent frequencies of each observed value. Students said that the graph was different from other graphs because it used the number line and Xs. No student said anything about being able to see gaps or holes in the data due to the use of a continuous scale.



The teacher decided to make sure that this aspect of the graph was noticed by students, so he asked very directly: “What does the number line help us see--something we haven’t seen so far from other displays?” A student answered that the inventors put numbers in between so she could see how far they went.

The teacher went on to emphasize how the display helped everyone notice gaps and holes in the measurements. Later (in Unit 3), this idea of distance between values came in handy, when students started to think about variability.

Bring this portion of the conversation to a close by asking:

Q: What are the three best displays for helping us see what the real length of my arms might be? Why?

*Note:* If no display uses a class interval (similar grouped values), introduce the notion of developing “bins” of similar values (see sample student work on page 11) by making one yourself. Students should also compare at least one display that uses a scale, because scales show holes in the data that can be obscured by class intervals, such as “the 160’s.”

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

## Instruction

## Inventing Displays Unit 1

### 3. Highlight the relationship between the shape of the data and process.

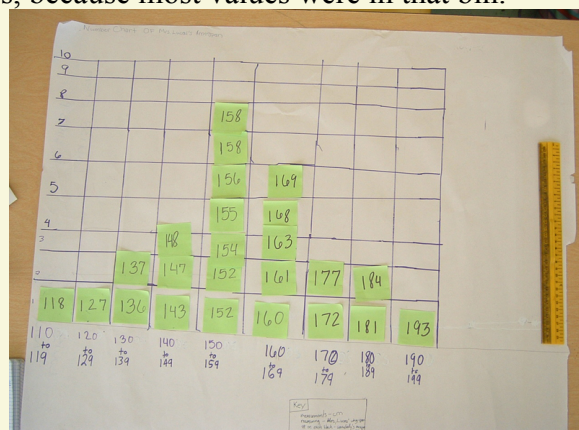
Your task is to help students relate data to the processes that generate them. This helps students experience data as meaningful. It helps set up the important idea of a repeated process, an idea that is foundational to measurement and to data analysis.

- Students work in pairs (or whole-group) to explain any patterns they see in a grouped values (interval) display or any other display that makes the center clump of the data visible.
- Use questions to generate discussion. For example:
  - Q: Why didn't everyone get the same value? If they did, what would the display look like? Why do you think so?
  - Q: What is it about the process of measurement that produces what we see?
  - Q: If the measurements just happened by chance, what would the display look like? Why do you think so? *Note:* All the measurements would be equally likely, so the data would be more uniform—no peaks.

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

### Classroom Talk: Relating display to process

In this example, using a grouped-values displays, a teacher asked, “What is your best guess of the length of your teacher’s arm span? Where do you think it is?” Most students were drawn to the center clump evident in the grouped-values display, suggesting that the “real length” of the arm span was in the 150s, because most values were in that bin.



## Instruction

## Inventing Displays Unit 1

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

The teacher asked, “Why are we getting graphs that look like this (normal curve)? Some values are under 150s and some values are over 150s.” Students’ reasoning tended to focus on their perceptions of “mistakes” when measuring. For example, some students mentioned that they left gaps when they iterated (moved) the ruler, and others mentioned overlapping the rulers (see figure on next page).

The teacher asked students to consider each form of error and to judge whether or not it would lead to an overestimate of the length (overlaps) or an underestimate of the length (gaps). This helped students account for the shape of the data. Overestimates of length resulted from using the ruler to measure the same distance more than once (overlaps). Underestimates resulted from not measuring some of the length (gaps).

- c. Ask what will happen if we do it again or if other people measured the same object.

The emphasis on “measure again” is more important than it might seem at first glance. The aim is to help students see their measurements as an outcome of a process—one that can be repeated, again and again. This is an important idea for thinking about chance (Unit 5). It also helps students view a measurement as composed of signal (the real length of the attribute) and error (mistakes, chance gaps and laps, etc.).

Have students predict what would happen if the measurement process were repeated (or if they measured again). Ask:

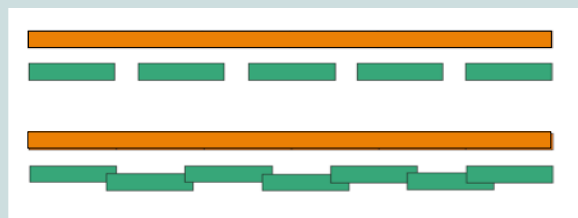
- Q: Suppose we could not remember our previous measurement. What would happen if we measured again (the same way using the same tool)? Would we get exactly the same measurement? Why do you think so?

## Instruction

## Inventing Displays Unit 1

### Students' Ways of Thinking: What will happen if we do it again?

Students typically believe that measurements will not be copied or exactly reproduced. Many suggest a tendency for future measurements to be located “in the middle clump.” Some students suggest that the extreme values will not be repeated, if one avoids errors of laps and gaps with the ruler.



*First image shows gaps. Second image shows laps.*

For example, a student suggested that students who produced measures of 170s or 180s would be more careful the second time, so they would get measurements closer the center of the data the second time, 140s, 150s, or 160s. The same student said that students who underestimated the first time would also tend to measure more carefully, so their measurements would more likely be in the middle bins of the data. But being “more careful” alters the process, so the teacher pressed students to consider what would happen if “we did it again but had no memory, or if other people measured? Would all the measurements be exactly the same?”

A few students may believe that if measurers were careful enough, they would all obtain the same value. (If we measure with no error, then the measurements will be identical.)

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

**Formative Assessment**

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

1. **Administer the Unit 1 quiz** (see page 24 or download from [Modelingdata.org](http://Modelingdata.org)).

*Jumping rope.* A conversation around this item is an opportunity to advance student thinking to consider how different ways of thinking about the data (such as attention to case values vs. attention to classes of similar values) result in different shapes for the same data. Students should compare and contrast some of these ways of thinking to promote reasoning about the data as an aggregate.

*Statue.* This item seeks to assess students' ability to recognize the effect of using different graphical representations of the same data. Students can show their understanding by comparing the two graphs of the measured heights of the statue. The two graphs in the item differ in that the histogram in Display A groups the measured heights of the statue into bins of 2 (e.g. 54-55, 66-67) while Display B only contains a single value per bin. In addition, Display B only displays values for which a measurement was taken and does not leave space for values that have no frequency. Hence, Display B's scale is not constant and skips the values in between 55 and 60.

2. **Score student responses using the Jumping Rope and Statue scoring guides.**
3. **Use Jumping Rope responses to generate a discussion of aggregate reasoning.**
  - a. **Select four student responses for students to compare and contrast.**

While scoring, select four different responses to use in the conversation. These responses should include one from each level on the scoring guide.

- b. **Prepare questions to guide and support student thinking.**

Examples of questions to use during the conversation:

- What does this display show about the data?
- What does this display not show or hide?



## Instruction

## Inventing Displays Unit 1

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

- (When Comparing) Which display would you use if you wanted to know the number of jumps that happened the most?
- (When Comparing) Which of these displays hides (pick a quality)? Does the other one show this more clearly?

**c. Use an Assessment Conversation to help students consider important elements of the displays.**

Invite students to present their responses. Guide the conversation with questions that direct the students to important elements of the displays. For example, when comparing a DaD 4(a) response with a DaD 3(a) response, it is important for students to see the work done by a constant scale. Notice that although DaD is used to score the responses, the students develop MRC reasoning when they are called upon to compare and contrast displays.

**d. Use an Assessment Conversation to help students consider how displays can help us make inferences about a question.**

Invite students who presented or ask the class at large to consider how the displays, and which displays, help us decide whether or not Dora's father was right.

**4. Use *Statue* responses to generate a discussion of the tradeoffs of different representations.**

**a. Select student responses to compare and contrast.**

While scoring, select two responses from each question. One example should only indicate what one of the displays shows, while the other should compare what the displays show and hide about the data.

**b. Prepare questions to guide and support student thinking.**

Examples of questions to use during the conversation:

- What aspects of each display help show the information needed in this question?
- Does one display show it better than the other? Why?
- Does either display hide (pick a quality mentioned in the question)?



**Instruction****Inventing Displays Unit 1**

- Are there situations where binning would be more useful than not binning?
  - Are there situations where not binning would be more useful?
  - Should a constant scale always be used? Why?
- c. **Use an Assessment Conversation to help students notice the importance of considering not only what displays show, but also what they hide.**

Compare and contrast as students present their responses. For questions 1 and 2 highlight the effects of binning on knowing how many students got each value. For Questions 3 and 4 highlight the use of the bins in emphasizing the concentration of values in the 60s while Display B is useful for showing more detail within this decade.

For example, the most frequent measurements were 65 and 66. This cannot be determined from Display A. For questions 5 and 6 highlight the effects of scale on the shape of the data. For questions 7 and 8 highlight the coordinated effects of binning and scale on estimating the actual height of the statue.

Taking Measurements  
Inventing Displays  
Comparing Displays  
Formative Assessment

## Unit Quiz

## Inventing Displays Unit 1

**Name:** \_\_\_\_\_

**Grade:** \_\_\_\_\_

**Teacher:** \_\_\_\_\_

**Gender: Male (boy)** \_\_\_\_\_ **Female (girl)** \_\_\_\_\_

**Language you speak at home:** \_\_\_\_\_

**Unit Quiz****Inventing Displays Unit 1****Jumping Rope**

Dora counted how many rope jumps she can do in one minute. Here is the number of jumps she did in 20 trials of one minute each.

25, 26, 27, 27, 26, 28, 30, 26, 27, 28, 26, 25, 27, 29, 28, 19, 26, 25, 28, 29

1. Given this sample, make a display that helps you think about how you expect Dora to perform in general.

Later, Dora's father gave her a lightweight jumping rope. He suggested that this rope will help her make more jumps in one minute. Dora counted her jumps with the lightweight rope. Here are the results of her 20 trials.

27, 28, 29, 29, 28, 30, 29, 28, 29, 30, 28, 29, 29, 30, 29, 29, 27, 30, 27, 28

2. Make a display that helps you think about Dora's performance using the lightweight rope.
  
  
  
  
  
  
  
  
  
  
3. Use the two displays you've created to make an argument for whether Dora's father was right or whether the differences between the data sets happened by chance. Be sure to use information from your displays in your argument.

---

---

---

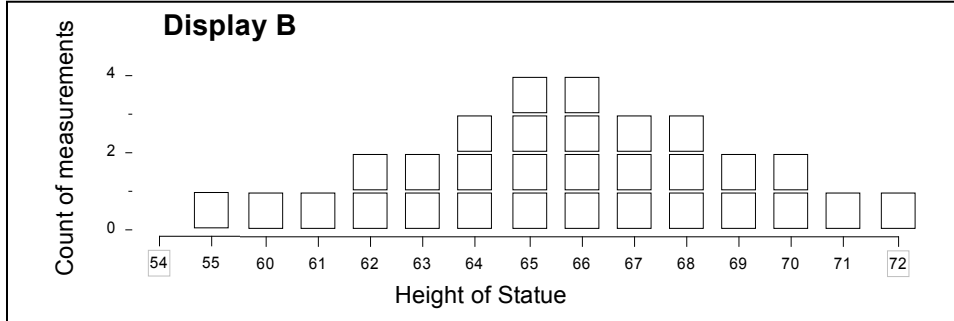
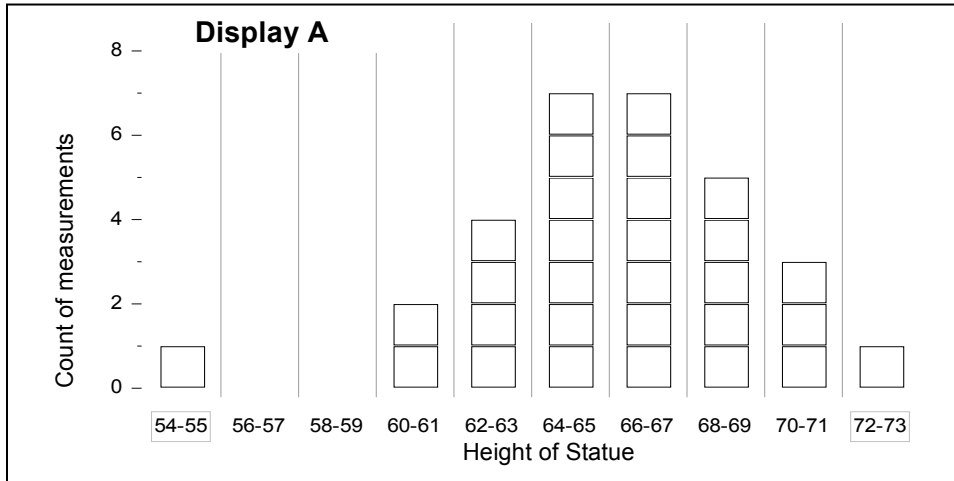
---

# Unit Quiz

# Inventing Displays Unit 1

## Statue

30 students measured the height of a statue in front of the city hall. Two students created displays of the data they collected. The two displays show the same data set.



- Which display does a better job at showing how many students got each value?
  - Display A
  - Display B
- Why is this display better than the other?

## Unit Quiz

## Inventing Displays Unit 1

3. Which display does a better job at showing where most of the measurements are?
  - a. Display A
  - b. Display B
  
4. Why is this display better than the other?
  
  
  
  
  
  
  
  
  
  
5. Which display does a better job at showing unusual measurements?
  - a. Display A
  - b. Display B
  
6. Why is this display better than the other?
  
  
  
  
  
  
  
  
  
  
7. We want to know the actual height of the statue. Which display is better for this purpose?
  - a. Display A
  - b. Display B
  
8. Why is this display better than the other?

# Unit Quiz Scoring Guide: Jumping Rope

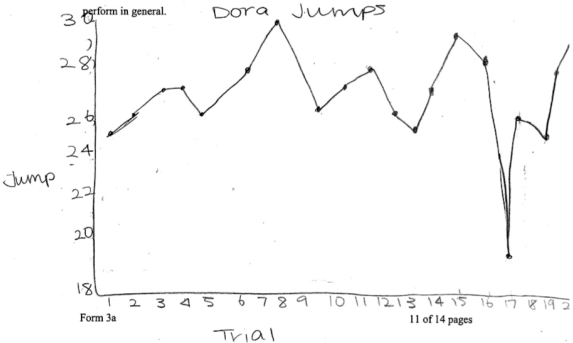
# Inventing Displays Unit 1

## Jumping Rope, Question 1

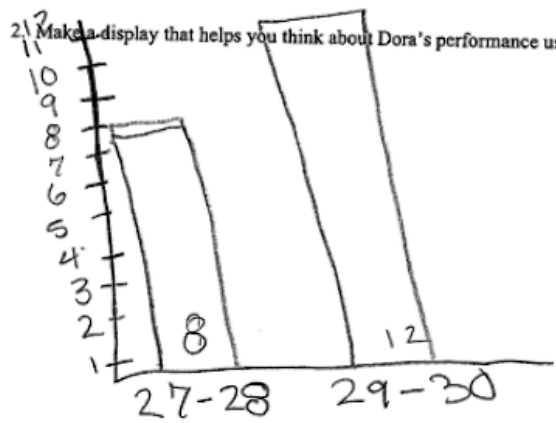
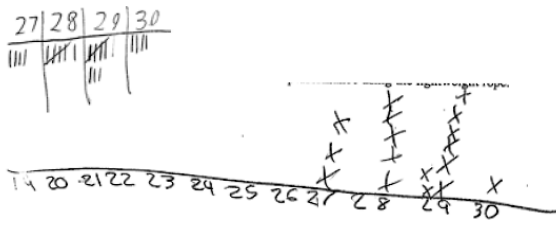
Question 1: Jumping Rope and Data Display (DaD)		
Level	Performance	Example
DaD(4a)	<p><b>Display data in ways that use its continuous scale (when appropriate) to see holes and clumps in the data.</b></p> <p>For the x-axis, the student must have continuous scale from 19-30. Students at this level should have BOTH <i>order</i> and <i>grouping</i>.</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b></li> </ul>
DaD(3a)	<p><b>Notices similar values or clumps in the data.</b></p> <p><b>GROUPING, NO CONTINUOUS SCALE (at a glance, the display shows relative frequencies):</b></p> <p>Displays are made by grouping identical values (such as in a frequency display) or making bins. Students do <i>not</i> have to order the groups.</p> <p>Displays must show the structure of the data. Displays that show frequency but in a way that misrepresents the data's structure are NOT scored at this level. This includes, for example, displays that show the "25's" column larger than the "26's" column.</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b> Student creates a display without a continuous scale (shown below)</li> </ul> <ul style="list-style-type: none"> <li><b>Example 2:</b> Student groups (by making a frequency display) but does not write a scale on the y-axis (shown below)</li> <li><b>Example 3:</b> Student groups with tally marks</li> </ul>

# Unit Quiz Scoring Guide: Jumping Rope

# Inventing Displays Unit 1

DaD(2b)	<p><b>Student manipulates data attending only to its ordinal properties.</b></p> <p><b>ORDER (at a glance, the display shows order but NOT relative frequencies):</b></p> <p>Student <i>orders</i> the data but does not use space to highlight aspects of the data's structure.</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b> Student puts the numbers in order but nothing additional*</li> <li><b>Example 2:</b> Student makes a frequency display but the display distorts the relative frequency. For example, at a glance, it appears that there are more 25's than 26's.*</li> </ul>
DaD(1a)	<p><b>Student manipulates, notices, or explores qualities or relations of data values, without relating to the question. Preliminary level of understanding is demonstrated.</b></p> <p>Here, if student attempts to use order or grouping, it does so in a way that distorts relative frequencies AND hides the frequency (see Example 3)</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b> Student employs an impractical method for binning such as "grouping evens and odds".</li> <li><b>Example 2:</b> Student treats each trial as change over time.</li> </ul> 
NL(ii)	<p>Response (graphical or otherwise) is at least somewhat relevant, but is unclear, or a restatement of given information.</p>	
NL(i)	<p>Response (graphical or otherwise) is irrelevant to the question.</p>	<ul style="list-style-type: none"> <li>“Don’t get it.”</li> <li>“He did better.”</li> </ul>
M	<p>Missing Response</p>	

Jumping Rope, Question 2

Question 2: Jumping Rope and Data Display (DaD)		
Level	Performance	Example
DaD(3a)	<p><b>Notices similar values or clumps in the data.</b></p> <p><b>GROUPING (at a glance, the display shows relative frequencies):</b></p> <p>Displays are made by grouping identical values (such as in a frequency display) or making bins. Students do <i>not</i> have to order the groups.</p> <p>Displays must show the structure of the data. Displays that show frequency but misrepresent the structure by using inconsistent markings, causing one value to inaccurately appear more frequent are NOT scored at this level. This includes, for example, displays that show the “28’s” column as larger than the 29’s” column. However, if the student uses consistent markings, but spacing between the markings is not consistent they will be scored 3A.</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b></li> </ul>  <ul style="list-style-type: none"> <li><b>Example 2:</b></li> </ul> 
DaD(2b)	<p><b>Student manipulates data attending only to its ordinal properties.</b></p> <p><b>ORDER (at a glance, the display shows order but NOT relative frequencies):</b></p> <p>Student orders the data but uses inconsistent markings that misrepresent the structure of the data and do not communicate the relative frequencies of the measures.</p>	<ul style="list-style-type: none"> <li><b>Example 1:</b></li> </ul> <p>27, 27, 27, 28, 28, 28, 28, 28, 29, 29, 29, 29, 29, 29, 29, 30</p>



## Unit Quiz Scoring Guide: Jumping Rope

## Inventing Displays Unit 1

DaD(1a)	<p><b>Student manipulates, notices, or explores qualities or relations of data values, without relating to the question.</b></p> <p>Demonstrates preliminary level of understanding of what a data display is.</p> <p>Here, if student attempts to use order or grouping, it is in a way that distorts relative frequencies AND hides the frequency</p>	<ul style="list-style-type: none"> <li>• <b>Example 1:</b> Student treats the values as change over time.*</li> </ul>
NL(ii)	Response (graphical or otherwise) is at least somewhat relevant, but is unclear, or a restatement of given information.	<ul style="list-style-type: none"> <li>•</li> </ul>
NL(i)	Response (graphical or otherwise) is irrelevant to the question.	<ul style="list-style-type: none"> <li>•</li> </ul>
M	Missing response	<ul style="list-style-type: none"> <li>•</li> </ul>

\*Mock student responses

# Unit Quiz Scoring Guide: Jumping Rope

# Inventing Displays Unit 1

## Jumping Rope, Question 3

The third question asks students to make an inference based on their displays. We will consider the Informal Inference (InI) construct in more detail later. For now, we suggest distinguishing among higher level and lower level responses.

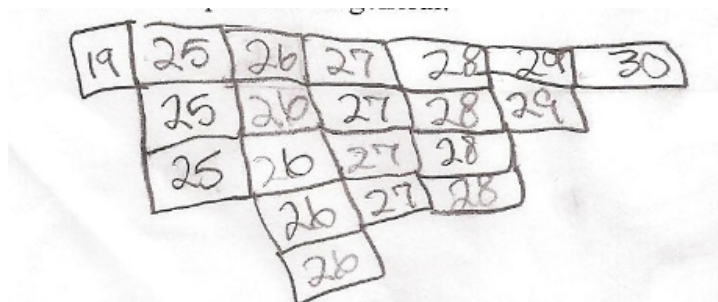
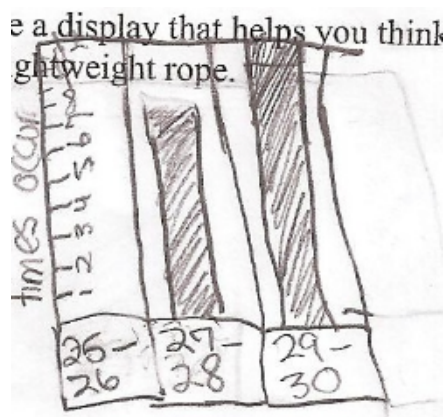
### Higher Level

Students anchor their recommendation to the displays and consider clusters of values. Typical student responses at this level:

#### Example 1H:

“He was right because with the lighter jump rope she got 29 and 30 12 times and when she used the regular rope, she had only 3 29 and 30.”

Note that this student created 2 different looking displays but both showed regions of values (DaD 3). This student’s displays were:



#### Example 2H:

“I think Dora’s father was right because with a heavier rope, her most common number of jumps, 26, was 5 times. With a lighter rope, her most common number of jumps was 29, and it was performed 8 times.” (Note: The modes are compared, which involves thinking about the frequencies, a kind of thinking about similar values. Later in instruction, we will expect students to consider a statistic of center in light of sample variability, but for now, this is a high level response.)

## Unit Quiz Scoring Guide: Jumping Rope

## Inventing Displays Unit 1

### Medium Level

**Example 1M:**

“Dora’s father was right because with the first rope, she jumped only 19 times in one minute, but with the lightweight rope, her lowest jump was 27 times in a minute.”

**Example 2M:**

“They are the same. You get 30 times in a minute with both ropes.”

**Example 3M:**

“They both go up and down.” (This student’s displays showed unordered case values.)

### Lower Level

Students do not anchor their recommendation to either display.

**Example 1L:**

“Lightweight ropes are easier to skip with.”

**Example 2L:**

“Dora should listen to her father.”

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

## Statue , Questions 1 and 2

Questions 1 and 2: How many students got each value?		
Level	Performance	Example
Correct	Display B	
Incorrect	Display A	
NL(i)	Student circles both answer choices or writes an irrelevant comment	<ul style="list-style-type: none"> <li>“I don’t know what this means.”</li> </ul>
M	Student does not circle an answer choice and does not write anything.	

Questions 1 and 2: Statue and meta-representational competence (MRC)		
Level	Performance	Example
MRC(4b)	<p><b>Student coordinates qualities of multiple displays with what those displays show and hide about the data.</b></p> <p>Student recognizes that display B shows the frequency for single values while display A doesn’t differentiate between the range of two values per bin.</p> <p>Student must mention or describe binning (or lack of binning) explicitly in one display.</p> <p>Student may mention how the other display does something that makes identifying individual values unclear</p>	<ul style="list-style-type: none"> <li>Display B, “because in Display A you can’t tell how many measurements they got for each height. In B, it shows how many measurements for each height.” *</li> <li>B, because A bins the measurements.</li> <li>Display B, “Display A has a number through another number. Display B has an exact number.”</li> <li>Display B, “cause the height of data in display B got one number for example 54 in display A got 2 number for example 54-55.”</li> <li>Display B, “because, Display A goes by 2s and you don’t know how many people go to what number</li> <li>Display B, “because the other one you can’t tell whether it is 60-61 or 54-55.”</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

<p><b>MRC(3a)</b></p>	<p><b>Compare displays by indicating what each shows about the structure of the data.</b></p> <p>Student describes what <i>one or both</i> of the displays show about the data relevant to the question (reasonable inferences can be made about relevance) but doesn't articulate what the displays hide. Specifically, students note the differences in <i>what the display shows</i> regarding individual values.</p> <p>Student may discuss that Display B shows each measurement explicitly, but if the process of binning (as in what the creator of the display did to ensure each measurement would be shown explicitly) is mentioned or described, then score as 4(b)</p>	<ul style="list-style-type: none"> <li>• Display B, “because it shows for each height, how many time it happened.”*</li> <li>• Display B, “because it shows exact number of kids”*</li> </ul>
<p><b>MRC(2b)</b></p>	<p><b>Student lists and compares observed characteristics of displays without explicit reference to data structure or purpose of data collection.</b></p> <p>Student compares observed characteristics of displays without explicit reference to data structure or purpose of data collection. **Note: “Bigger” is interpreted as focusing on the x-axis of the display.</p>	<ul style="list-style-type: none"> <li>• Display A, “because it is more bigger there the other one”</li> <li>• Display A, “because <i>it is shorter</i> and gives more information that we need.”</li> <li>• Display A, “because it has a title and labels.”*</li> <li>• Display A, “because instead of skipping the numbers that aren't being used, they still have them”</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

<b>NL(ii)</b>	<p><b>Student's response is somewhat relevant to the problem but unclear.</b></p> <p>Student is clearly referring to the display but more specific referents may not be clear.</p> <p>This also includes responses where the student has identified non-existent features of the display (for instance, saying Display A shows the values 50-51).</p>	<ul style="list-style-type: none"> <li>• Display B, "Is because is easy to understand what they are doing"</li> <li>• Display B, "looks like it has all the right measurements than display A"</li> <li>• Display B, "cause the other is hard to explain"</li> <li>• Display A, "Display A makes more sense than Display B"</li> <li>• Display A, "It shows you it more clear."</li> <li>• Display B, "It doesn't just not give a lot of information. Display A didn't have much."</li> <li>• Display A, "Because it doesn't skip numbers."*</li> </ul>
<b>NL(i)</b>	Response is irrelevant, unclear, or a restatement of given information.	<ul style="list-style-type: none"> <li>• Display A, "because it is the best to right"</li> <li>• "What do you mean?"</li> </ul>
<b>M</b>	Missing response Student provides no explanation.	<ul style="list-style-type: none"> <li>• Display B (no explanation)</li> <li>• Display A (no explanation)</li> </ul>

\*Mock student responses

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

## Statue, Questions 3 and 4

Questions 3 and 4: Statue and Data Display (DaD)		
Level	Performance	Example
DaD(3a)	<p><b>Student notices or constructs groups of similar values from distinct values.</b></p> <p>Student either: chooses display A and provides a reason why the grouping of the values is useful for finding, <i>OR</i> chooses display B and provides a reason why exact values are preferable over groupings. <i>OR</i> choose either by noting the clump.</p>	<ul style="list-style-type: none"> <li>• Display B, “because, Display A has (example) on numbers 64-65, people 7 people voted, how do you know how many people got 64 instead of 65”</li> <li>• Display A, “It is because display B shows how many people got each exact thing while display a just rounds together in groups, making it easier to see were the biggest and smallest measurements are.”</li> <li>• Display A, “it shows that 2 of the bars are way higher than all of the other bars.”</li> <li>• Display A, “There more 64-65 and 66-67”</li> </ul>
DaD(2a)	<p><b>Student concentrates on specific data points without relating these to any structure in the data.</b></p> <p>Student may choose Display B, noting that the values are written individually, but does NOT evaluate the advantages for this structure compared to binning. Students who do weigh the advantages are scored as DaD3(a).</p> <p>Students who refer to a grouping without further elaboration are also included (see last two examples).</p>	<ul style="list-style-type: none"> <li>• Display B, “Only two measurements were exactly the same.”</li> <li>• Display B, “because 64 and 65 have the most.”*</li> <li>• Display B, “because they are written one by one”</li> <li>• Display B, “because they are written one by one”</li> <li>• Display A, “Because it combines 64 and 65 and 7 people said that.”</li> </ul>
DaD(1a)	<p><b>Student interprets data displays or describes features of the display without relating to the goals of the inquiry.</b></p> <p>Any response that interprets the display without clearly addressing the goals of the inquiry</p>	<ul style="list-style-type: none"> <li>• “I counted and they both have the same number of people.”*</li> <li>• “Display B because it has 14 groups.”*</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

NL(i)	<p><b>Response did not explicitly mention features of the display and/or interpret data.</b></p> <p>Any responses that did not interpret the data using information explicit in the display.</p>	<ul style="list-style-type: none"> <li>• Display A, “I think because it got more measurements than the other.”</li> <li>• Display A, “most of them are on a line so you can tell”</li> <li>• Display A, “it shows more.”</li> <li>• Display B, ”because it has more information”</li> <li>• Display A, “the dots are closer together.”</li> <li>• “It has a key.”</li> <li>• “it is good and perfect”</li> <li>• Display A, “because it is higher”</li> </ul>
M	Missing response	<ul style="list-style-type: none"> <li>• Display A, (no explanation)</li> <li>• Display B, (no explanation)</li> </ul>

\*Mock student responses



## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

## Statue, Questions 5 and 6

Questions 5 and 6: Unusual measurements		
Level	Performance	Example
Correct	Display A	
Incorrect	Display B	
NL(i)	Student circles both answer choices or writes an irrelevant comment	<ul style="list-style-type: none"> <li>• “I don’t know.”*</li> </ul>
M	Student does not circle an answer choice and does not write anything.	

Questions 5 and 6: Statue and Meta-representational Competence (MRC)		
Level	Performance	Example
MRC(4b)	<p><b>Student coordinates qualities of multiple displays with what those displays show and hide about the data.</b></p> <p>Student recognizes that display A shows the holes in the data while display B doesn’t.</p>	<ul style="list-style-type: none"> <li>• Display A, “because the x-axis of the display has bins although the bins do not have any measurements in them. However, Display B only marks measurements on the x-axis that measurements exist.” *</li> </ul>
MRC(3a)	<p><b>Student compares displays by indicating what each shows about the structure of the data.</b></p> <p>Students may answer either Display A or Display B and discuss what one or each shows about the data but does not discuss what they hide about the interpretation of the data.</p>	<p><b>Students notes hole in Display A:</b></p> <ul style="list-style-type: none"> <li>• Display A, “because the x-axis of the display has bins that do not have any measurements in them.”*</li> <li>• Display A, “There is 4 measurements between 54-55 and 60-61.”</li> <li>• Display A, “Because there are none on some.”</li> <li>• Display A, “because display B skips 56 - 59”*</li> </ul>
MRC(2b)	<p><b>Student lists and compares characteristics of displays without explicit reference to data structure or purpose of data collection.</b></p>	<ul style="list-style-type: none"> <li>• Display B, “because its smaller”</li> <li>• Display B, “it is longer”</li> <li>• Both displays used squares to show</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

	<p>Student makes a correct interpretation or observation that is not relevant to the question.</p> <p><b>**Note:</b> “Bigger” is interpreted as focusing on the axis of the display.</p>	<p>measurements. *</p> <ul style="list-style-type: none"> <li>• Display, “B they all look low and about the same”</li> <li>• Display A, “All of them are different heights”</li> <li>• Display A, “because it goes one number to another. Example 1-5”</li> <li>• B, “because it doesn’t have all the numbers.”</li> </ul> <p><b>Student may notice what a display shows or hides, but if the student misinterprets “unusual measurements”, then the response is scored as 2(b). For instance, the following two responses consider A to be the better display because the binning hides where the measurements go:</b></p> <ul style="list-style-type: none"> <li>• Display A, “This display is better because it shows heights in intervals of 2 so we don't know which height goes where.”</li> <li>• Display A, "It shows 3 or 4 at a time"</li> </ul> <p><b>Students mention binning or grouping measurements together.</b></p> <ul style="list-style-type: none"> <li>• Display B, “because it has each different measurement.”</li> <li>• B, “because it shows each individual measurement.”*</li> </ul>
NL(ii)	<p>Student’s response is somewhat relevant to the problem but unclear or without an explanation.</p> <p>This includes responses where the student has identified non-existent features of the display (for instance, saying Display A shows the values 50-51).</p> <p>Includes responses where student’s logic</p>	<ul style="list-style-type: none"> <li>• Display B, “because its an unusual way of showing the measurements by little measurements of the height”</li> <li>• Display A, “it gives less information.”</li> <li>• Display B, “because it is easier to explain.”</li> <li>• Display B, “because it is not that</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

	does not match between multiple choice and explanation.	accurate.” • “Display B, because it bins the values.”*
NL(i)	Response is irrelevant, unclear, or a restatement of given information.	• “because it’s just right.”* • “I don’t know”*
M	Missing response	• Display A, (no explanation) • Display B, (no explanation)

\*Mock student responses

Questions 5 and 6: Statue and Data Display (DaD)		
Level	Performance	Example
DaD4(a)	<p><b>Student displays data in ways that use its continuous scale (when appropriate) to see holes and clumps in the data.</b></p> <p>Student chooses display A because display B does not have a constant scale.</p>	<ul style="list-style-type: none"> <li>• Display A, “because display B skips 56 - 59” *</li> <li>• Display A, “because display A shows all values even if there are no boxes” *</li> <li>• Display A, “because it shows 54-55 way farther then the others”</li> <li>• Display A, “Because there are none on some.”</li> </ul>
DaD3(a)	<p><b>Student notices or constructs groups of similar values from distinct values.</b></p> <p>Student identifies possible unusual values.</p> <p>Student may choose Display B and provide a reason why exact values are preferable over groupings.</p>	<ul style="list-style-type: none"> <li>• Display B, “because on Display A shows 72-73 you can't know if it's a 72 or 73”</li> <li>• Display B, “Because on A you can’t tell if its 54 or 55” *</li> </ul>
DaD2(a)	<p><b>Student concentrates on specific data points without relating these to any structure in the data.</b></p> <p>Student may choose Display B, noting that the values are written individually, but does NOT evaluate the advantages for this structure compared to binning.</p>	<ul style="list-style-type: none"> <li>• Display B, “because it shows each height.”*</li> <li>• Display B, “because it shows what each student measured.”</li> </ul> <p><b>The following two responses consider A to be the better display because the binning hides where the</b></p>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

	<p>Students who do weigh the advantages are scored as DaD3(a).</p> <p>Student may also choose Display A and concentrate on particular points that are not relevant to highlighting how the structure of the display shows unusual measurements.</p>	<p><b>measurements go (a noticing irrelevant to how the display's structure highlights unusual measurements):</b></p> <ul style="list-style-type: none"> <li>• Display A, "This display is better because it shows heights in intervals of 2 so we don't know which height goes where."</li> <li>• Display A, "It shows 3 or 4 at a time"</li> </ul>
DaD1(a)	<p><b>Student interprets data displays or describes features of the display without relating to the goals of the inquiry.</b></p> <p>Any response that interprets the display without clearly addressing the goals of the inquiry</p>	<ul style="list-style-type: none"> <li>• Display B, "all of the measurements is there on the graph"</li> <li>• Display B "its more spread apart"</li> <li>• Display A, "because it counts by twos and doesn't skip numbers"</li> <li>• "I counted and they both have the same number of people."*</li> <li>• "Display B because it has 14 groups."*</li> </ul>
NL(i)	<p><b>Response did not explicitly mention features of the display and/or interpret data.</b></p> <p>Any responses that did not interpret the data using information explicit in the display.</p>	<ul style="list-style-type: none"> <li>• Display A, "because it is the one that is write." (uninformative)</li> <li>• Display A, "All of them are different heights" (not true)</li> <li>• Display B, "because its smaller" (unclear what "its" refers to)</li> <li>• Display, "B they all look low and about the same" (not true)</li> <li>• Display A, "because it goes one number to another. Example 1-5" (unclear)</li> <li>• Display A, "it gives less information." (not true)</li> </ul>
M	Missing response	<ul style="list-style-type: none"> <li>• Display A, (no explanation)</li> <li>• Display B, (no explanation)</li> </ul>

\*Mock student responses

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

## Statue, Questions 7,8

## Questions 7 and 8: Actual height of the statue / Meta-representational Competence (MRC)

NOTE: Depending on students' reasoning, either A or B can be chosen. Because of this, there is no coding of "correct" or "incorrect" responses.

Level	Performance	Example
MRC(4b)	<p><b>Student coordinates qualities of multiple displays with what those displays show and hide about the data.</b></p> <p>Students may either answer A or B as being better for showing the actual height of the statue.</p> <p>Students must mention what one of the displays did to show or hide something about the data.</p> <p>Student may recognize that display B shows the frequency for single values while display A doesn't differentiate between the range of two values per bin.</p> <p>Student may recognize that display A shows a tighter clump of values around the center.</p>	<ul style="list-style-type: none"> <li>• Display B, "it shows what each person got and helps estimating the greatest measurement while on display A, you can't figure out how many people got an answer."</li> <li>• Display A, "because it's easier to see where most of the measurements are and that's probably where the actual height of the statue is."*</li> </ul>
MRC(3a)	<p><b>Student compares displays by indicating what each shows about the structure of the data.</b></p> <p>Students may answer either Display A or B and discuss what either or both displays show but do not discuss what they hide.</p> <p>Students may refer to "measurement" or "measurements" rather than "height" or "heights."</p> <p>In contrast to 2(b) where students may misinterpret the meaning of "actual height" to mean "individual height," students may mention individual heights in the context of what the display shows (one shows individual measurements,</p>	<p><b>Student notes that Display B shows the frequency for each statue height:</b></p> <ul style="list-style-type: none"> <li>• Display B, because it has the numbers by itself. (example: 55, 54, 56, 57)</li> <li>• Display B, because it's not like all in one measurement and that one has its in different measurements.</li> <li>• Display B, because A gives you whole lot number to work with. B give you certain numbers.</li> <li>• Display B, because it goes one by one.</li> <li>• Display B, it shows what most</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

	one does not) but it is not clear that they misinterpreted the meaning of the question.	<p>people got.</p> <p><b>Student notes that Display A bins the data</b></p> <ul style="list-style-type: none"> <li>• Display A, it shows more people agreeing to one thing.</li> <li>• A, “because it tells where most of the measurements were.”</li> <li>• B, “because in A you can’t see each person’s measurement.”</li> </ul>
MRC(2b)	<p><b>Student lists and compares characteristics of without explicit reference to data structure or purpose of data collection.</b></p> <p><i>**Note:</i> “Bigger” is interpreted as focusing on the axis of the display.</p> <p>This includes responses where a reasonable inference can be made that the student has misinterpreted “actual height” to mean each student’s measurement being shown individually. Student may refer to “height” or “heights” rather than “measurement” or “measurements.”</p>	<ul style="list-style-type: none"> <li>• Display A, “because it is taller than B. B is shorter than A.”</li> <li>• Display B, “because it has more width and height.”</li> <li>• Display B, “because the height of the statue is in order from least to greatest.”</li> <li>• Display B, because you can see the heights. (<i>Note:</i> This is an example of a student misinterpreting “actual height” to mean each measurement shown individually.)</li> </ul>
NL(ii)	<p>Student’s response is somewhat relevant to the problem but unclear.</p> <p>This includes responses where the student has identified non-existent features of the display (for instance, saying Display A shows the values 50-51).</p>	<ul style="list-style-type: none"> <li>• Display A, because it probably is tall.</li> <li>• Display A, if you look at it is no doubt on which one is the actual height.</li> <li>• (The student interprets the display incorrectly): Display A, because it shows every single height from 54 to 73</li> <li>• Display B, “because it is easier to explain.”</li> <li>• Display A, because you can see the heights. (<i>Note:</i> This is a misinterpretation, since B shows the individual heights)</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

NL(i)	Response is irrelevant, unclear, or a restatement of given information.	<ul style="list-style-type: none"> <li>• Display B, “because it is a good thing to do.</li> <li>• Display B, “because it has more display than A.”</li> <li>• Display A, “because it is the write answer.”</li> </ul>
M	Missing response	<ul style="list-style-type: none"> <li>• Display A, (no explanation)</li> <li>• Display B, (no explanation)</li> </ul>

Questions 7 and 8: Statue and Data Display (DaD)		
Level	Performance	Example
DaD(5a)	<p><b>Student recognizes that a display provides information about the data as a collective.</b></p> <p>Student can choose either display and provides a reasonable explanation identifying the center clump of data as the best estimate for the actual height.</p>	<ul style="list-style-type: none"> <li>• Display B, “because on the graph it shows that it should be either 65 or 66 because they have the same number of blocks.”</li> <li>• Display A, “it shows that most measurements are between 64-67. So its easy to guess what is the real height.” *</li> </ul>
DaD(3a)	<p><b>Student notices or constructs groups of similar values from distinct values.</b></p> <p>Student chooses a display based upon identifying a clump of data.</p> <p>Student may also choose Display B and provide a reason why exact values are preferable over groupings.</p>	<ul style="list-style-type: none"> <li>• Display A, “I think this one because the heights are more tall.”</li> <li>• Display A, “because it reaches to 7”</li> <li>• Display A, “It shows more people agreeing to one thing”</li> <li>• Display A, “Because it goes all the way up to 7 and the other dose not. It stops at 4.”</li> </ul>
DaD(2a)	<p><b>Student concentrates on specific data points without relating these to any structure in the data.</b></p> <p>Student chooses a display based upon specific cases.</p> <p>Student may choose Display B, noting that the values are written individually, but does NOT evaluate the advantages</p>	<ul style="list-style-type: none"> <li>• Display B, “because it shows that 65 and 66 have the most.”*</li> <li>• Display B, “Because it shows every single height from 54 to 73.”</li> <li>• Display B, “cause it's not like all in one measurement and that one has it in different measurements.”</li> </ul>

## Unit Quiz Scoring Guide: Statue

## Inventing Displays Unit 1

	for this structure compared to binning. Students who do weigh the advantages are scored as DaD3(a).	<ul style="list-style-type: none"> <li>• Display B, “because it shows 64, 65 not 64-65 like A does.”</li> <li>• Display B, “it tells you the exact measurements”</li> </ul>
DaD(1a)	<p><b>Student interprets data displays or describes features of the display without relating to the goals of the inquiry.</b></p> <p>Any response that interprets the display without clearly addressing the goals of the inquiry</p>	<ul style="list-style-type: none"> <li>• “I counted and they both have the same number of people.”*</li> <li>• “Display B because it has 14 groups.”*</li> <li>• Display B, “all of the measurements is there on the graph”</li> <li>• Display B, “because they are in order”</li> <li>• Display A, “it is clustered”</li> <li>• Display A, “Because it is taller.”</li> </ul>
NL(i)	<p>Response did not explicitly mention features of the display and/or interpret data.</p> <p>Any responses that did not interpret the data using information explicit in the display.</p>	<ul style="list-style-type: none"> <li>• “What do you mean?”</li> <li>• Display A, “because it is the write answer.” (uninformative)</li> <li>• Display A, “because it probably is tall.” (unclear what “it” refers to)</li> <li>• Display A, “if you look at it is no doubt on which one is the actual height.” (unclear)</li> <li>• Display B, “Because it is more presifix than the other one.” (jibberish)</li> <li>• Display B, “because it has more with and height.” (not true)</li> </ul>
M	Missing response	<ul style="list-style-type: none"> <li>• Display A, (no explanation)</li> <li>• Display B, (no explanation)</li> </ul>