John von Neumann’s Life and Achievements

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Theory of Calculus

Childhood

- John von Neumann was born on December 28th, 1903, in Budapest, Hungary into a wealthy family.
- Neumann reportedly had an eidetic memory, allowing for an absolute recall of everything he read.
- As a child he learned Ancient Greek, Calculus, and the history of several wars and empires.

Early Education

- John von Neumann was homeschooled until he was eleven years old. This was typical in Hungary at the time.
- He then attended the Budapest-Fasori Evangélikus Gimnázium. He attended this school at the same time as three famous physicists: Leo Szilard, Eugene Wigner, and Edward Teller.

University

- Neumann originally enrolled in the University of Berlin to pursue a two-year non-degree seeking course in Chemistry due to an agreement with his father.
- However, due to his interest in mathematics, he also enrolled in University Pázmány Péter in Budapest as a Ph.D. candidate.
- His Ph.D. thesis was on the axiomatization of Cantor’s Set Theory.
- Neumann completed his Ph.D. courses in absentia, only appearing to take exams due to his studies in Berlin.

University

- While in Berlin, von Neumann attended statistical physics lectures taught by Albert Einstein.
- He also collaborated with Erhard Schmidt on Set Theory while in Berlin.

University

- John von Neumann first studied Hilbert's theory of consistency with German mathematician Hermann Weyl.
- He eventually graduated both as a chemical engineer from ETH and with Ph.D. in mathematics, summa cum laude from the University of Budapest in 1926.

Göttingen

- Neumann applied to continue his studies in mathematics, with recommendations from Richard Courant, Hermann Weyl, and David Hilbert.
- With such impressive recommendations and his qualifications, Neumann was accepted to the University of Göttingen and began his studies in 1926 under the world-famous mathematician David Hilbert.
- Drawn to Hilbert because of David Hilbert’s contributions to Formalism, Neumann worked under Hilbert from 1926 until 1930.

Set Theory

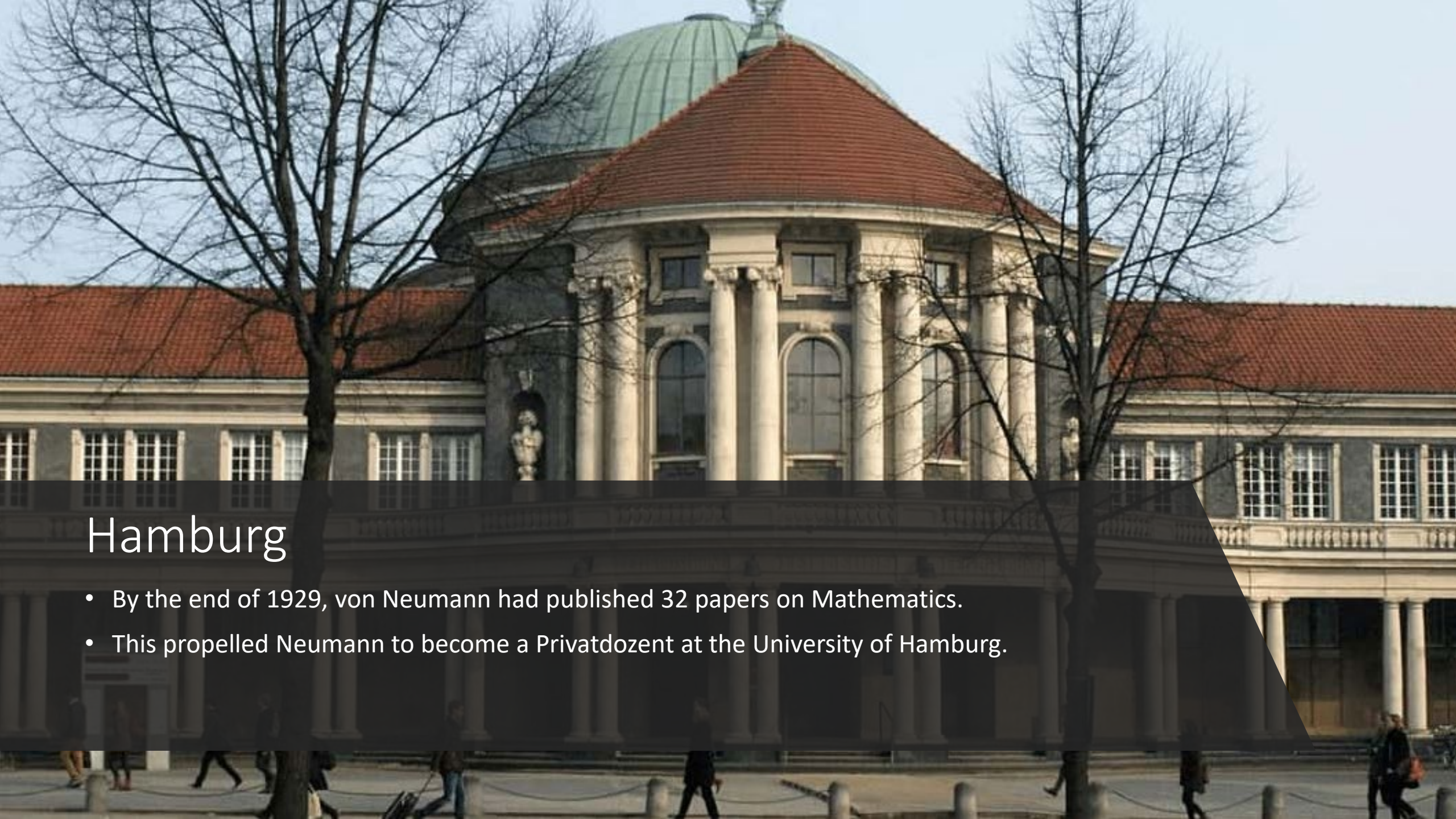
- Between 1923 and 1929, von Neumann wrote six papers on Set Theory.
- His main contribution to Set Theory was the von Neumann-Bernays-Gödel set theory which is essentially an extension of the Zermelo-Fraenkel set theory.
- This contribution introduced the notion of class to Set Theory.
- Neumann’s work provided solutions to some of the problems of Zermelo set theory, leading to the eventual development of Zermelo-Fraenkel set theory.

End of Neumann’s work in Formalism

- Kurt Gödel’s incompleteness theorem effectively ended von Neumann and David Hilbert’s work on formalism.
- Kurt Gödel's incompleteness theorem is within the scope of the mathematical philosophy of Platonism. Given that Platonism is a theory contrary to Formalism, this ended the work of Hilbert and Neumann and forever changed the philosophy of mathematics.

Game Theory

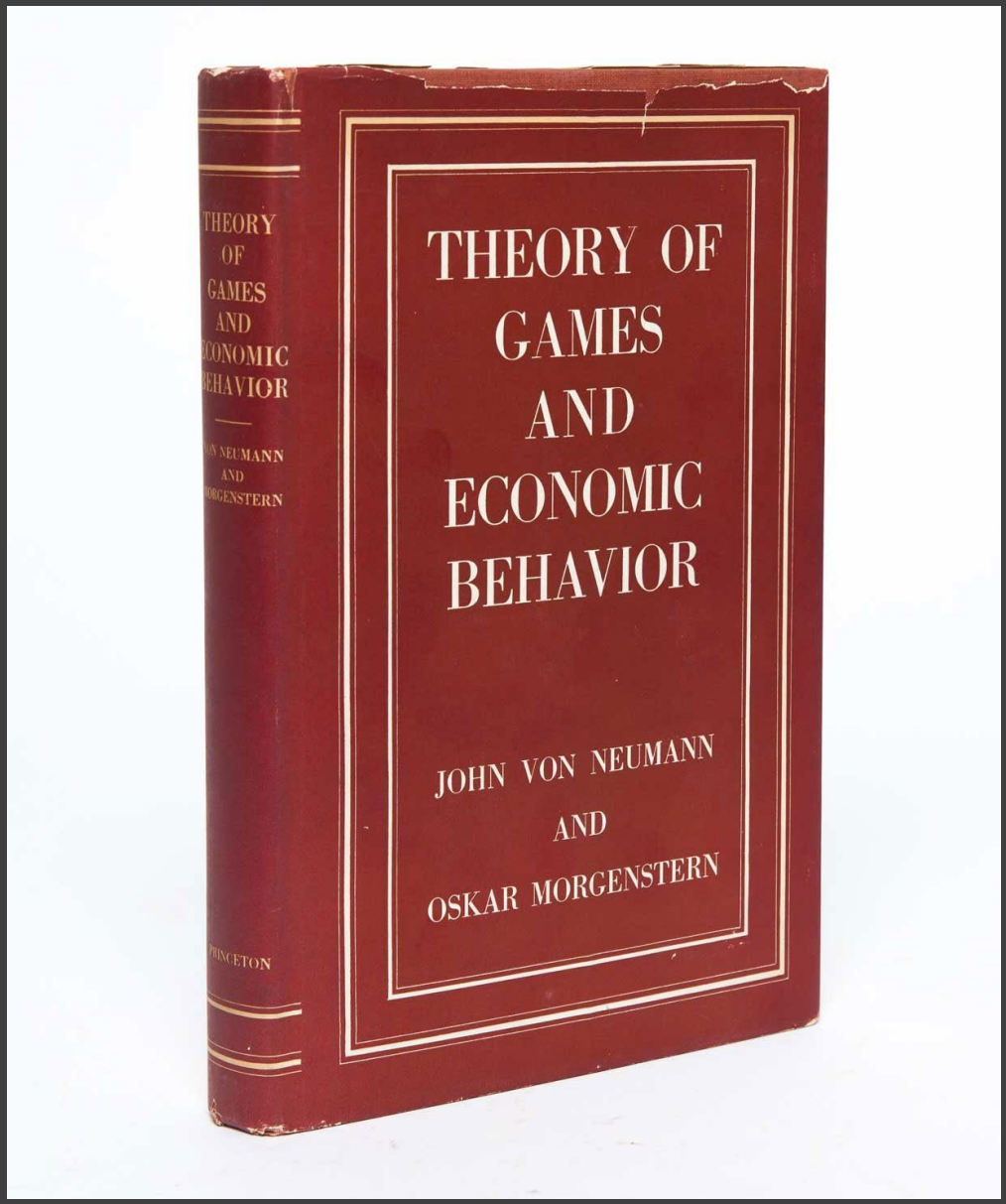
- By 1927 Neumann had published 12 papers on mathematics, leading to his appointment as a *Privatdozent* at the University of Berlin in 1928 at the age of 25.
- Around the same time von Neumann was making contributions to set theory, von Neumann also proved a theorem known as the minimax theorem for zero-sum games, which would later lay the foundation for the new field of game theory as a mathematical discipline.
- The minimax theorem provides the conditions that guarantee that the max-min inequality is also an equality, that every finite, zero-sum, two-person game has optimal mixed strategies.



Hamburg

- By the end of 1929, von Neumann had published 32 papers on Mathematics.
- This propelled Neumann to become a Privatdozent at the University of Hamburg.

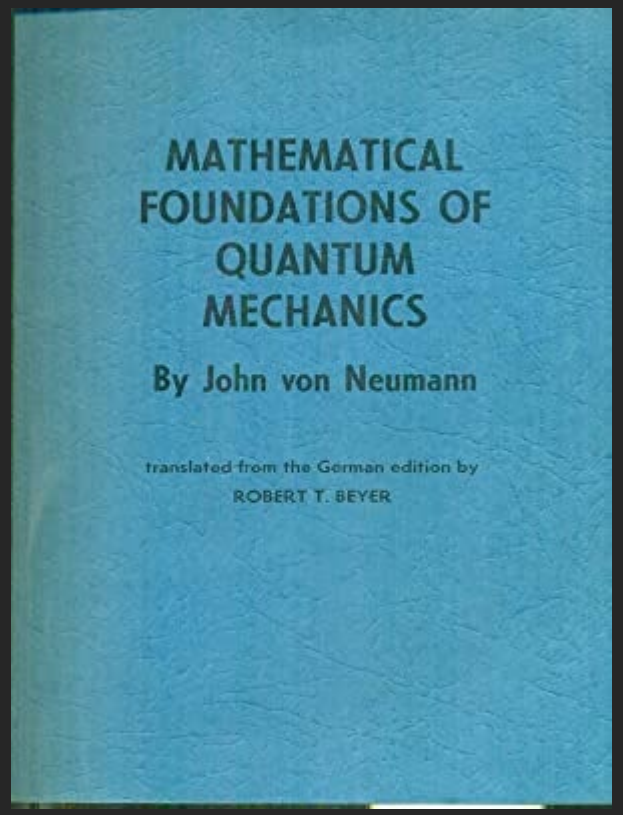
Theory of Games and Economic Behavior- Published in 1944



Quantum Mechanics

- Later in his life, von Neumann himself stated that his most important work in Quantum mechanics was his work done in 1926 at Göttingen and his work done from 1927-1929 in Berlin.
- Neumann eventually came to establish a rigorous mathematical foundation for Quantum Mechanics in the form of the Dirac-von Neumann axioms.

Quantum Mechanics



- From 1927 to 1931 von Neumann wrote 5 papers which each constituted a notable contribution to Quantum Mechanics.
- Neumann essentially established the mathematical framework for Quantum Theory and contributed to the statistical aspects of Quantum Theory.
- His work on Quantum Mechanics was published in his 1932 book “Mathematical Foundations for Quantum Mechanics”.

Operator Theory

- John von Neumann studied set theory, quantum mechanics, and algebra while he was in Berlin.
- In the field of algebra, he particularly studied Operatory theory which can be defined as the study of linear operators on function spaces, beginning with differential operators and integral operators.
- He invented von Neumann Algebra which is a *-algebra of bounded operators on a Hilbert space that is closed in the weak operator topology and contains the identity operator.

Princeton

- John von Neumann received an invitation to become a visiting professor at the University of Princeton, which he accepted. He began working there in 1930 and left in 1933.
- His work at Princeton was focused on Ergodic Theory which is the branch of mathematics that studies the statistical properties of deterministic dynamical systems.
- Neumann wrote two papers in the scope of Ergodic Theory while at Princeton: Proof of the Quasi-ergodic Hypothesis (1932) and Physical Applications of the Ergodic Hypothesis (1932).

Princeton

- His work on Ergodic Theory led him to create the “von Neumann’s mean Ergodic Theorem” which essentially provided a mathematical basis for the statistical mechanics of liquids and gases.
- The von Neumann’s mean Ergodic Theorem:

For any isometric operator U on a Hilbert space H and for any $h \in H$ the limit

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=} 0^{n-} 1 U^k h = \overline{h}$$

exists (in the sense of convergence in the norm of H). For a continuous one-parameter group of unitary transformations $\{U_t\}$ on H and any $h \in H$, the limit

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T U_t h dt = \overline{h}$$

exists (in the same sense). Here $\overline{h}$ is the orthogonal projection of h onto the space of U- (or $\{U_t\}$-) invariant elements of H.

Institute for Advanced Study

- After von Neumann left Princeton University in 1933, he accepted a professorship at the Institute for Advanced study.
- Because the University was founded in 1930, von Neumann was one of the first six professors there alongside Albert Einstein, J. W. Alexander, Marston Morse, Oswald Veblen, and Hermann Weyl.

Continuous Geometry

- While at the Institute for Advanced Study, von Neumann published two papers in 1936.
- The first paper established Continuous Geometry as a field of Mathematics, and the second provided examples of its use.

Service in the United States Army

- In 1937, certain that a war was coming, John von Neumann decided to join the United States Army as a lieutenant in the reserve of the ordnance department of the U.S. Army.
- von Neumann did this because he was interested in accessing statistics regarding explosions. This is because the science of explosions was an area in which he developed expertise.

Los Alamos and the Manhattan Project

- John von Neumann was not one of the original scientists tasked with working on the Manhattan project, although he did pay multiple visits to Los Alamos as a sort of consultant.
- Due to his expertise in explosions, eventually von Neumann did work on explosive lenses which were required for the construction of the “Fat Man” nuclear bomb which was dropped on Nagasaki.
- He also made discoveries which led to the United States government’s decision to detonate their Atomic Bombs a few kilometers above their target rather than on impact.

Computing

- After his work at Los Alamos, von Neumann saw the need for more computing power in order to make the large amounts of calculation required for the successful building and detonation of an atomic bomb more efficient.
- Neumann eventually proposed the von Neumann architecture which is a type of computer architecture designed to process arithmetic logic.
- He also created the “merge sort algorithm” and introduced stochastic computing.

Personal Life and Consultancies

- John von Neumann was a consultant for several different government organizations as well as private companies once he moved to the United States.
- He enjoyed listening to loud German music in his office, reading while driving, and partying.
- von Neumann was diagnosed with what was likely a form of cancer which confined him to a wheelchair at 51 years old.
- John von Neumann died February 8th, 1957, at 53 years old.
- He is now buried at Princeton Cemetery in New Jersey alongside his friends Eugene Wigner and Kurt Gödel.

Sources

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