Compact acoustic bandgap material based on a subwavelength collection of detuned Helmholtz resonators

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This study explores the amplitude and phase transmission of audio-frequency sound through a waveguide side-loaded with a series of closely spaced and sequentially tuned Helmholtz resonators. This system exhibits a series of acoustic bandgaps due to the Helmholtz resonance and standing-wave cavity modes. The bandgaps are achieved in a physically compact manner in that the resonators are spaced by much less than a wavelength. The response of the Helmholtz resonator-loaded waveguide is described by a simple adaptation of an existing theory. Finally, within the forbidden transmission regions the system exhibits narrow bands of negative group delay. © *2011 American Institute of Physics*. [doi: 10.1063/1.3595677]

I. INTRODUCTION

In this paper we report experimental and theoretical analyses of audio-frequency sound wave propagation in an acoustic waveguide side-loaded with a series of closely spaced, and sequentially detuned, Helmholtz resonators. The objectives of the paper are twofold. First, we demonstrate the ability to design and engineer acoustic bandgaps in a compact (less than a wavelength) arrangement of resonators. Second, we analyze the properties of this system as a potential acoustic metamaterial.

Acoustic bandgaps,[1] and their electromagnetic counterparts photonic bandgaps,[2,3] are typically observed in periodic systems where the coherent effects of scattering and interference lead to frequency bands over which transmission is forbidden. As the mechanism of bandgap formation is based on a number of periodic scattering elements each spaced by about a quarter wavelength, the physical dimension of a bandgap material is typically much larger than the wavelength at the fundamental bandgap frequency. The physical size of bandgap systems can be detrimental in many vaunted applications of these materials. A number of systems to realize more compact bandgap materials have been reported. For electromagnetic radiation, frequency-selective surfaces have been explored as highly reflective substrates for antennas[4] or the uniplanar compact photonic bandgap structures for microwave circuit applications.[5] In the acoustic realm there has been strong interest in fabricating artificial materials that can manipulate the properties of the sound field. One-dimensional gratings[6] and concentric corrugated bulls-eye structures[7] exhibit enhanced acoustic transmission and collimation similar to extraordinary optical transmission.[8] The key difference between the acoustic and optical realms is that, in the optical case, surface plasmons are widely cited as mediating the enhanced transmission. There is no corresponding acoustic excitation to surface plasmons, so the explanations for the enhanced acoustic transmission invoke diffractive and Fabry–Pérot resonances instead. It is interesting to note that the Helmholtz resonance exploited in the experiments described here is a uniquely acoustic phenomenon with no optical counterpart. Another approach has been to use plates periodically perforated with subwavelength hole arrays.[9] This system exhibits extraordinary acoustic transmission at certain resonant frequencies in addition to acoustic screening over a wide wavelength range. Strong acoustic attenuation also has been achieved through the use of mass-loaded membranes.[10] The vibrational resonances of membranes can be used to create very strong attenuation of sound, particularly at low frequencies. Yang *et al.*[10] showed that low frequency sound attenuation 200 times the mass density value could be obtained with suitably designed membranes. The work described here is similar to that of Ref. 10 in that we use a tunable resonator—in our case a Helmholtz resonator cavity instead of a mass-loaded membrane—to realize acoustic attenuation in a one-dimensional waveguide. Here we report the observation of forbidden transmission bands in an acoustic waveguide loaded with a series of sequentially detuned Helmholtz resonators. The resonators are closely spaced physically along the waveguide in a distance much less than the wavelength of sound at the forbidden transmission band frequencies. We show that this simple system exhibits fundamental and higher order bandgaps and that our experimental observations are described by a simple extension of an elegant theoretical formalism for Helmholtz-loaded waveguides presented by Wang *et al.*[11]

The second issue explored in this work is whether the system of detuned Helmholtz resonators demonstrates the characteristics of an acoustic metamaterial. Recently, a model acoustic metamaterial has been described based on the use of subwavelength-spaced *identical* Helmholtz resonators side-loaded on an ultrasonic waveguide.[12] Fang *et al.*[12] showed that this system exhibits acoustic bandgaps

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and that in the bandgap region the material shows the characteristics hallmark of a metamaterial; negative group delay of tunneled pulses indicating an antiparallelism of the group and phase velocities. Here we show that the use of detuned, rather than identical, resonators significantly changes the phase behavior within the bandgap such that there is not a broad frequency range of negative-group delay across the whole gap. Rather, there are only narrow frequency intervals of negative group delay within the gap associated with the individual resonances of each of the Helmholtz resonators. This finding shows that the detuned resonators behave essentially independent of each other despite their physical proximity, whereas the results of Ref. 12 are a consequence of the collective interaction of the resonators with identical frequencies.

II. EXPERIMENTAL DETAILS

Experimentally, we first characterized the response of a waveguide side-loaded with a single Helmholtz resonator cavity. This configuration has been explored theoretically by Wang *et al.*[11] We show that our single Helmholtz resonator results agree well with the theoretical model. Through a simple extension of the theoretical model we designed a system of four sequentially detuned Helmholtz resonators that exhibits broad forbidden transmission gaps. The experimental results are in good agreement with this simple extension of the single Helmholtz resonator theoretical model in both amplitude and phase response.

In order to measure experimentally the transmission function of the resonator-loaded waveguide over a broad frequency range, an add-and-average impulse response method was used as outlined in Ref. 13. The setup is shown schematically in Fig. 1(a). The left and right stereo outputs of a sound card in a personal computer were used, respectively, to generate an audio impulse and a trigger signal. The impulse was routed to an audio amplifier connected to a speaker. The speaker was attached to one end of the waveguide system under test. At the other end of the waveguide the transmitted impulse was recorded by a microphone (ACO Pacific 7022). Simultaneously, the trigger signal was sent to a universal serial bus connected data-acquisition module (IOTech PersonalDAQ 3000) that initiated analog-to-digital conversion of the signal from the microphone. The signal from the microphone was recorded by the computer. The entire process of pulse transmission, reception, and recording was controlled by a MATLAB program.

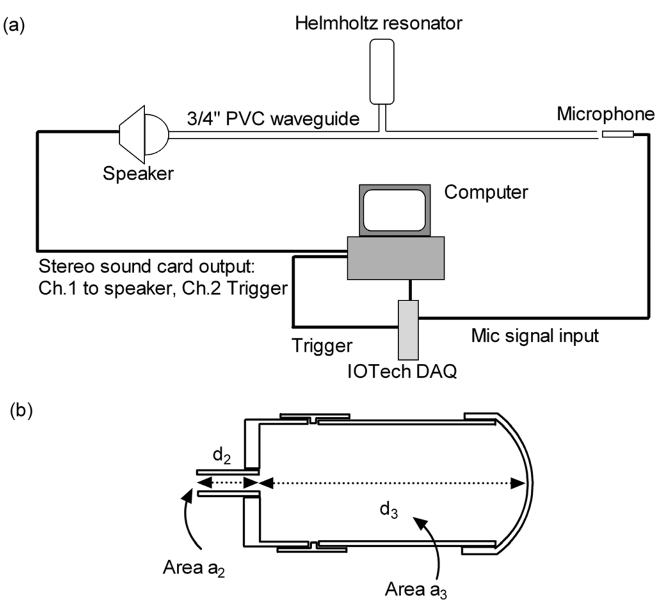

FIG. 1. (a) Schematic of experimental setup. (b) Details of a single Helmholtz resonator.

The impulse signal used in these experiments was the second derivative of a Gaussian function. This pulse shape is well replicated by speakers and provides a broad audio spectrum from 100 to 3000 Hz.[13]

The add-and-average technique was used to acquire impulse response data with very high signal to noise. Instead of determining the response due to a single impulse, a typical run consisted of about 100 impulses that would be averaged to obtain the final response signal. Each recorded impulse signal was added to the previous and averaged in order to cancel any random ambient noise. The MATLAB control program generated the trigger/impulse pair at random intervals to decrease any cyclical system noise.

To determine the response of a loaded waveguide we first acquired a reference signal of the impulse propagated through a long straight section (18.3 m) of waveguide with no loading. The acoustic waveguide used in these experiments was standard 3/4 in. (1.9 cm) polyvinyl chloride (PVC) plumbing pipe. A Helmholtz resonator or array of resonators was then placed in the center of the 18.3 m waveguide. When the resonator sample was inserted into the waveguide enough PVC pipe was removed to ensure that the physical distance from the speaker to microphone was the same as for the reference. The Helmholtz resonators were made from standard PVC plumbing components: the neck was 3/4 in. pipe, a 3/4–2 in. adapter transitioned to 2-in.-diam. pipe for the body of the resonator, and a 2-in.-diam. end cap. The resonator structure is shown schematically in Fig. 1(b). A 3/4 in. T junction was used to connect the neck of the resonator to the waveguide. The experiment was then repeated, with the impulse being sent down the loaded waveguide. Both the experimental and reference signals were time windowed to eliminate comb filtering due to reflections at the ends of the waveguide. Both time signals were Fourier transformed to convert the experimental data into the frequency domain. The complex transmission function was then calculated by dividing the Fourier transform of the experimental signal by that of the reference. The magnitude of the complex transmission function represents the normalized amplitude transmission through the resonator-loaded system as a function of frequency. The phase angle of the complex transmission at each frequency represents the relative phase change introduced by the sample under test.

III. RESULTS OF LOADING BY A SINGLE RESONATOR

Experimental results for the transmitted amplitude and phase due to loading by a single Helmholtz resonator with dimensions of $a_2 = 3.1$ cm^2, $d_2 = 4.5$ cm, $a_3 = 20.4$ cm^2,

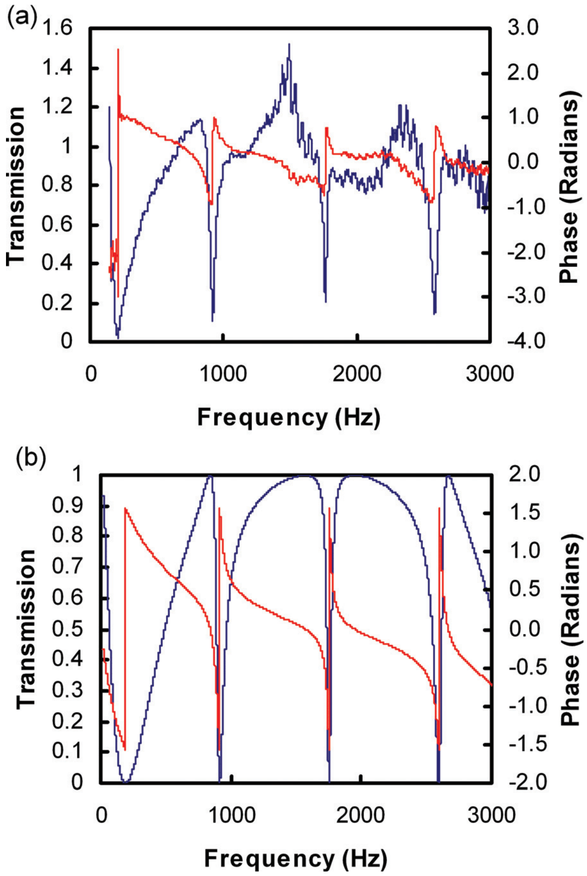

FIG. 2. (Color online) (a) Experimental amplitude transmission (lower trace) and phase (upper trace) as a function of frequency for a single Helmholtz resonator. (b) Theoretical amplitude transmission (lower trace) and phase (upper trace) based on the theory of Wang *et al*. (Ref. 11).

$d_3 = 20.9$ cm [dimension labels defined in Fig. 1(b)] are shown in Fig. 2(a). The amplitude transmission [lower trace in Fig. 2(a)] shows dips at 208, 920, 1770, and 2590 Hz. The phase response [upper trace in Fig. 2(a) referenced to the right-hand axis] demonstrates a distinct positive phase jump coincident with each transmission minimum.

It is easy to assign the lowest frequency transmission dip to the Helmholtz resonance frequency given by

$$f_{HR} = \frac{c}{2\pi}\sqrt{\frac{A}{V_0 L}}, \tag{1}$$

where A is the area of the neck opening, L is the neck length, c is the speed of sound, and V_0 the enclosed volume of the resonator. Using the above-given dimensions for the resonator used to obtain the data in Fig. 2(a) the Helmholtz resonator frequency is 219 Hz, in reasonable agreement with that measured experimentally. However, to understand the data more fully we made use of an existing theoretical formalism described next.

The experimental arrangement used to acquire the transmission and phase data shown in Fig. 2(a) corresponds precisely to the theoretical system modeled by Wang *et al.*[11] Using an interface response formalism the authors in that work derived the following expression for the complex transmission, t, of a waveguide side-loaded with a single Helmholtz resonator:

$$t = \frac{\cot(\alpha_2 d_2) - \frac{Z_2}{Z_3}\tan(\alpha_3 d_3)}{\cot(\alpha_2 d_2) - \frac{Z_2}{Z_3}\tan(\alpha_3 d_3) + \frac{i}{2}\frac{Z_1}{Z_2}\left[1 + \frac{Z_2}{Z_3}\cot(\alpha_2 d_2)\tan(\alpha_3 d_3)\right]} \tag{2}$$

The waveguide cross-sectional area is a_1, the neck area, a_2, and the cavity area, a_3. The neck and cavity lengths are given by d_2 and d_3, respectively. The Z_i values are the corresponding acoustic impedances of each tubular section given by $Z_i = \rho c/a_i$, where ρ is the density of air and $\alpha_i = \omega/c$. Using MATLAB, the expression for t was calculated as a function of frequency using the dimensions appropriate to our Helmholtz resonator. The theoretical amplitude and phase data are displayed in Fig. 2(b). There is generally excellent agreement between the theory and experiment both in amplitude and phase. Because the theoretical model assumes no loss the dips in transmission are deeper and the phase jumps at the transmission minima steeper in the theoretical model than in the experimental data.

There are two particular features of note from the results on the single Helmholtz resonator. First, a Helmholtz resonator is a simple harmonic oscillator and should therefore exhibit only a single resonant frequency. Clearly here there are a number of resonances. It is easy to show by a simple simulation using Eq. (2) that the higher order resonances correspond to standing-wave modes along the length of the cavity. If the area of the cavity, a_3, is doubled while the cavity length, d_3, is halved, then the enclosed cavity volume remains constant and the Helmholtz resonator frequency should be unchanged. A simulation of this arrangement shows that the lowest transmission minimum remains at exactly the same frequency, whereas the second transmission dip (at 860 Hz) disappears and only the next highest one is present. This result occurs because the shorter cavity has a higher standing-wave frequency by a factor of 2. The second feature of note is the characteristic π-phase jump at each of the transmission minima. A phase change on transmission is an unusual feature and does not occur in wave propagation between normal materials. However, as described by El Boudouti *et al.*[14] for the case of electrical transmission-line loop filters, these phase jumps can lead to narrow bands of strongly anomalous dispersion and negative group velocity wave propagation. Phase jumps in acoustic loop filters were used in the first demonstration of negative group velocity sound wave propagation.[15] It is clear that a waveguide side-loaded with a single Helmholtz resonator also demonstrates a similar strongly anomalous dispersion, and as is shown in the following, negative group velocity occurs over very narrow frequency intervals near the transmission minima.

IV. RESULTS OF LOADING BY MULTIPLE DETUNED RESONATORS

The next goal of our experimental investigation was to create a system that provides a broader frequency bandgap interval by side-loading the waveguide with a closely spaced ($<\lambda/5$) arrangement of Helmholtz resonators, each with slightly detuned frequencies. These frequencies were adjusted by changing the cavity length, d_3. They ranged

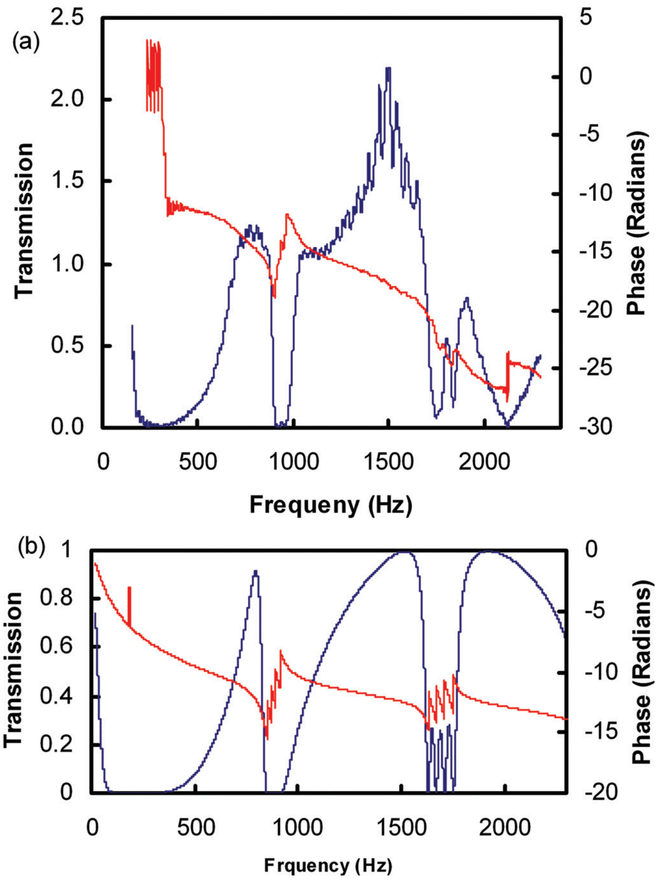

FIG. 3. (Color online) (a) Experimental amplitude transmission (lower trace) and phase (upper trace) as a function of frequency for four slightly detuned Helmholtz resonators. (b) Theoretical amplitude transmission (lower trace) and phase (upper trace) as a function of frequency for four slightly detuned Helmholtz resonators.

from $d_3 = 20.9$ to 19.4 cm, in four equal 0.5 cm increments. The amplitude and phase response measured experimentally for this system is shown in Fig. 3(a). The transmission amplitude data show a wide fundamental bandgap centered at 275 Hz, with a frequency width of 400 Hz. A second, narrower but sharper gap was achieved around ~900 Hz, with a width of 100 Hz. This corresponds quite well with the gaps predicted [Fig. 2(b)], and even the shape and behavior of these bandgaps is quite similar to the anticipated behavior for the given range of frequencies.

To extend the theory of Ref. 11 to describe a number of closely spaced Helmholtz resonators, we assumed that each Helmholtz resonator acts independently, i.e., one Helmholtz resonator does not affect the behavior of adjacent Helmholtz resonators. There is no *a priori* justification for this assumption, particularly in light of the results of Ref. 12 where closely spaced identical Helmholtz resonators apparently demonstrate collective behavior, or given previous work on the coupling between identical resonators.[16] In the end, the only validation of this assumption is that the experiment and theory agree well. Wang *et al.*[11] extended their theory to the case of periodically spaced Helmholtz resonators, in which case the bandgaps that open up are intrinsically associated with the periodic structure. In contrast to the work presented here, the spacing between Helmholtz resonators is less than $\lambda/5$ up to a frequency of about 1400 Hz.

With the assumption of independent Helmholtz resonators, the total amplitude transmission, t_T, through a series of i resonators each with individual transmission t_i is given simply by the product of the t_i values,

$$t_T = \prod_i t_i \tag{3}$$

Using the d_3 values for the four Helmholtz resonators used in our experiment (19.4, 19.9, 20.4, and 20.9 cm) results in the theoretically calculated transmission shown in Fig. 3(b). The agreement with the experimental amplitude transmission data presented in Fig. 3(a) is very good, even past the 1400 Hz limit where we might anticipate that periodicity effects begin to play a role on the transmission function. The lowest order bandgap due to the overlapping Helmholtz resonances is broad covering the interval from 180 to 500 Hz. This frequency range corresponds to wavelengths from 0.7 to 1.9 m from a set of resonators that occupy a total length of only 0.1 m along the waveguide. The second order gap centered at 950 Hz is also well defined. This gap is the result of overlapping standing-wave resonances along the length of the cylindrical cavities. The phase data are also in good agreement, except that the experimental low frequency phase data are noisy and inconsistent with the theory. There are two reasons why the low frequency phase data are not reliable: The impulse that we use has very weak frequency content at low frequency and there is a high pass filter in the data acquisition board that affects the low frequency phase data.

V. PHASE ANALYSIS AND GROUP DELAY

Finally, the phase data can be interpreted to show that negative group delays are present in narrow spectral regions within the forbidden transmission bands. The group delay time, τ_g, can be calculated from the frequency dependent phase data using

$$\tau_g = -\frac{\partial \varphi}{\partial \omega} \tag{4}$$

As the phase data in the lowest bandgap are not of sufficient quality, the group delay time is calculated for the second bandgap centered at 950 Hz. In Fig. 4 the group delay calculated by a numerical differentiation based on Eq. (4) is plotted as a function of frequency superimposed on the amplitude transmission for the second bandgap. The series of sharp drops in group delay correspond to the phase jumps in the transmission data. Negative group delays are on the order of 1 ms. The series of narrow intervals of negative group delay are associated with the phase jump associated with each individual Helmholtz resonator. This result contrasts with the result of Ref. 12 in which the identical Helmholtz resonators gave rise to a more smoothly varying phase and concomitant group delay. One of the hallmarks of a metamaterial is a group velocity antiparallel to the phase velocity.[12] Here, although there is a negative group delay, the concept of group velocity is difficult to define because the Helmholtz resonator occupies essentially no distance along the waveguide.

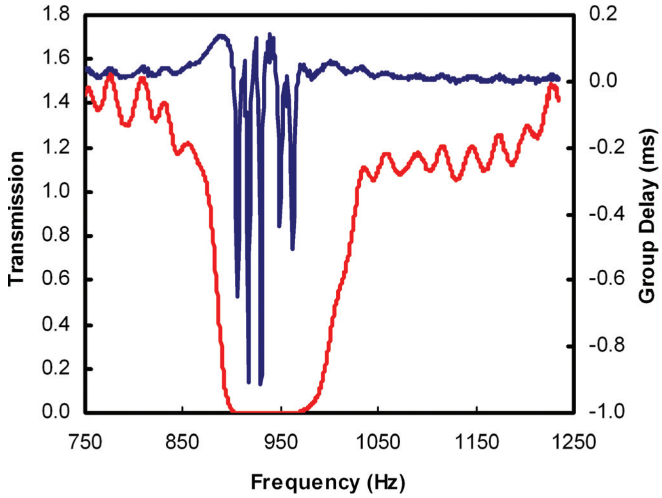

FIG. 4. (Color online) Plot of amplitude transmission (lower trace) and group delay (upper trace) for the second bandgap.

VI. CONCLUSION

In summary, we have demonstrated a sequentially detuned Helmholtz resonator system that permits the creation of wide acoustic bandgaps in a compact format without the need for periodicity. We showed that an existing theoretical model for a single side loading of a waveguide by a resonator[11] worked well to describe our single Helmholtz resonator data. A simple extension of this model to the case of a series of detuned Helmholtz resonators was similarly shown to give very good agreement with experiment. Finally, we observed that there is a characteristic phase jump, in both theory and experiment, coincident with the transmission minimum. We showed that the phase jumps lead to the existence of negative group delays, albeit over very narrow frequency intervals.

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